

The Easiest Way to Convert Units in an Algebraic Equation

Physics professors teach you to convert everything into standard SI units, solve the problem, and hope the units come out right. In Chemistry and the Engineering disciplines, we use the three-step **Factor-Label Method of Unit Conversion** to solve algebraic problems with mixed units; this method is more useful for real problems, because real objects often have mixed unit measurements...a 4 ft. x 8 ft. x 1/2 inch sheet of plywood; a 1 1/2 in. x 3 1/2 in. x 8 ft. board. Even metric steel I-beams have mixed units: depth in millimeters, length in meters, mass per unit length in kilograms per meter (which has to be converted into weight per unit length).

Follow these three easy steps to solve unit problems:

Step 1 Write the algebraic equation so the desired quantity is on the left of the equals sign, and an algebraic expression is on the right of the equals sign.

Step 2 Draw a horizontal line on the page, and enter numbers and units above and below the line according to the algebraic expression.

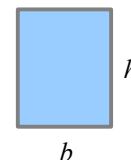
Step 3 Draw a vertical line to show the separation between each unit conversion, and enter all unit conversions necessary to solve the problem. If the unit is raised to a power, then the conversion factor **and** unit must be raised to that power. Consider memorizing the most common conversion factors, like the ones at the right. The units in the final answer **must** appear in the equation, **and** all other units must cancel.

1 ft.	=	12 in.
1 m	=	100 cm = 10 ³ mm
1 kip	=	10 ³ lb.
1 Pa	=	1 N/m ²

Metric prefixes		
milli-	(m)	= 10 ⁻³
centi-	(c)	= 10 ⁻²
kilo-	(k)	= 10 ³
Mega-	(M)	= 10 ⁶
Giga-	(G)	= 10 ⁹

Example #1: Area in U.S. Units

The area of a rectangle is $A=b \cdot h$. Given a base $b=83$ in. and a height $h=45$ ft., calculate the area in square feet.



Step 1 The algebraic equation does not need to be manipulated.

Step 2 Draw a horizontal line. Enter 83 in. and 45 ft. in the numerator.

$$A = b \cdot h = \frac{83 \text{ in.} \quad 45 \text{ ft.}}{\quad}$$

Step 3 We want to eliminate inches to obtain a final result in square feet. Therefore, put 12 inches in the denominator of the unit conversion, and 1 ft. in the numerator. Cross out duplicate terms, and the answer is 311.25 ft.²

$$A = b \cdot h = \frac{83 \text{ in.} \quad 45 \text{ ft.}}{\quad} \left| \frac{\text{ft.}}{12 \text{ in.}} \right.$$

$$A = b \cdot h = \frac{83 \cancel{\text{in.}} \quad 45 \text{ ft.}}{\quad} \left| \frac{\text{ft.}}{12 \cancel{\text{in.}}} \right. = 311.25 \text{ ft.}^2$$

The reason we can multiply by $\frac{\text{ft.}}{12 \text{ in.}}$ is that $1 \text{ ft.} = 12 \text{ in.}$, therefore $\frac{\text{ft.}}{12 \text{ in.}} = 1$. Unit conversions involve multiplying equations by 1, something you can always do without changing the result.

Example #2: Stress in SI Units

Stress is force divided by area. If the stress is 1 N/mm², what is the stress in MPa?

Step 1 There is no algebra to solve here because we are converting one unit to another.

Step 2 Draw a horizontal line. Enter 1 N in the numerator, and mm² in the denominator.

$$\frac{1 \text{ N}}{\text{mm}^2}$$

Step 3 A pascal is defined as Pa=N/m², so enter Pa in the numerator. Instead of writing in N/m² the denominator, put N in the denominator and m² in the numerator.

$$\frac{1 \text{ N}}{\text{mm}^2} \left| \frac{\text{Pa} \text{ m}^2}{\text{N}} \right.$$

Now enter the unit conversions to eliminate the two area terms: m² and mm². There are 10³ mm in a meter, so use parentheses to square the number and the unit.

$$\frac{1 \text{ N}}{\text{mm}^2} \left| \frac{\text{Pa} \text{ m}^2}{\text{N}} \right| \frac{(10^3 \text{ mm})^2}{\text{m}^2}$$

Finally, 1MPa=10⁶Pa. Put MPa in the numerator and 10⁶ Pa in the denominator. If you write the equation without numbers, it looks like $\frac{\text{N}}{\text{mm}^2} \left| \frac{\text{Pa} \text{ m}^2}{\text{N}} \right| \frac{\text{mm}^2}{\text{m}^2} \left| \frac{\text{MPa}}{\text{Pa}} \right.$. Cross out duplicate units, and all units cancel except for MPa. If you write the equation without units, it looks like $\frac{1}{\text{mm}^2} \left| \frac{(10^3)^2}{\text{m}^2} \right| \frac{1}{10^6}$.

$$\frac{1 \text{ N}}{\text{mm}^2} \left| \frac{\text{Pa} \text{ m}^2}{\text{N}} \right| \frac{(10^3 \text{ mm})^2}{\text{m}^2} \left| \frac{\text{MPa}}{10^6 \text{ Pa}} \right.$$

$$\frac{\text{N}}{\text{mm}^2} \left| \frac{\text{Pa} \text{ m}^2}{\text{N}} \right| \frac{(\text{mm})^2}{\text{m}^2} \left| \frac{\text{MPa}}{\text{Pa}} \right.$$

Solving the equation with numbers and units, we get 1 N/mm²=1 MPa . This is a useful conversion factor.

$$\frac{1 \text{ N}}{\text{mm}^2} \left| \frac{\text{Pa} \text{ m}^2}{\text{N}} \right| \frac{(10^3 \text{ mm})^2}{\text{m}^2} \left| \frac{\text{MPa}}{10^6 \text{ Pa}} \right. = 1 \text{ MPa}$$

Duplicate units in Examples #1 and #2 are crossed out; you can cross out duplicate units in the remaining Examples.

Example #3: Thermal Expansion in U.S. Units

Deflection due to thermal expansion is $\delta = \alpha \cdot L \cdot \Delta T$. The upper-case Greek letter delta means “change”, so ΔT means “change in temperature.” Given a deflection $\delta = 0.06 \text{ in.}$, a length $L = 8 \text{ ft.}$, and a thermal expansion coefficient $\alpha = 5 \times 10^{-6} \text{ }^\circ\text{F}^{-1}$, calculate the change in temperature in degrees Fahrenheit.

Step 1 Rewrite the equation algebraically to solve for ΔT .

$$\delta = \alpha \cdot L \cdot \Delta T$$

$$\Delta T = \frac{\delta}{\alpha L}$$

Step 2 Draw a horizontal line. Enter 0.06 in. in the numerator; enter $5 \times 10^{-6} \text{ }^\circ\text{F}^{-1}$ and 8 ft. in the denominator.

$$\Delta T = \frac{0.06 \text{ in.}}{5 \times 10^{-6} \text{ }^\circ\text{F}^{-1} \cdot 8 \text{ ft.}}$$

Since $^\circ\text{F}$ is to the power -1, write it in the numerator to the power +1.

$$\Delta T = \frac{0.06 \text{ in.}}{5 \times 10^{-6} \text{ }^\circ\text{F}^{-1} \cdot 8 \text{ ft.}}$$

Step 3 Convert feet to 12 inches so the length units to cancel, and the result is in $^\circ\text{F}$.

$$\Delta T = \frac{0.06 \text{ in.}}{5 \times 10^{-6} \text{ }^\circ\text{F}^{-1} \cdot 8 \text{ ft.}} \left| \frac{\text{ft}}{12 \text{ in.}} \right. = 125 \text{ }^\circ\text{F}$$

Example #4: Stress in U.S. Units

Stress is force divided by area: $\sigma = \frac{P}{A}$. Given a force $P = 7000$ lb. acting on an area $A = 3$ ft.², calculate the stress in units of pounds per square inch (psi).

Step 1 The equation does not need to be manipulated.

Step 2 Draw a horizontal line. Enter 7000 lb. in the numerator, and 3 ft.² in the denominator.

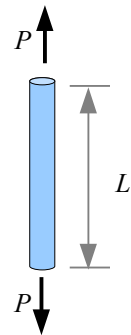
$$\sigma = \frac{P}{A} = \frac{7000 \text{ lb.}}{3 \text{ ft.}^2}$$

Step 3 The stress is in units of pounds per square foot. There are 12 inches in a foot, but we need to convert square feet, so square the number and the unit: (12 in.)². Square feet cancel, and the answer is in pounds per square inch, also written psi.

$$\sigma = \frac{P}{A} = \frac{7000 \text{ lb.}}{3 \text{ ft.}^2} \left| \frac{\text{ft.}^2}{(12 \text{ in.})^2} \right| = 16.2 \frac{\text{lb.}}{\text{in.}^2} = 16.2 \text{ psi}$$

Example #5: Deflection under Tensile Load in SI Units

A tensile bar stretches an amount $\delta = \frac{P \cdot L}{A \cdot E}$ where P is the applied load, L is the length of the bar, A is the cross-sectional area, and E is Young's Modulus. The bar has a circular cross section. Given a load of 30 kN, a length of 80 cm, a diameter of 6 mm, and a Young's Modulus of 207 GPa, calculate the deflection in mm.



Step 1 In math class, the area of a circle is given by $A = \pi r^2$. In real life, we measure diameter using calipers; it is much easier to measure a diameter than a radius on most objects. Convert radius to diameter, and the area equation becomes more useful. This is a good equation to memorize.

$$A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4} = \frac{\pi}{4} d^2$$

Combine the two equations to obtain a single algebraic equation.

$$\delta = \frac{P \cdot L}{A \cdot E} = \frac{4 P L}{\pi d^2 E}$$

Step 2 Draw a horizontal line and enter the numbers and units.

$$\delta = \frac{4 \cdot 30 \text{ kN} \cdot 80 \text{ cm}}{\pi (6 \text{ mm})^2 207 \text{ GPa}}$$

Step 3 The SI unit of stress or pressure is the pascal, where $\text{Pa} = \frac{\text{N}}{\text{m}^2}$, so $\text{GPa} = \frac{10^9 \text{ N}}{\text{m}^2}$. Since $1 \text{ kN} = 10^3 \text{ N}$, we can write $\text{GPa} = \frac{10^6 \text{ kN}}{\text{m}^2}$. Three conversion factors are needed: one to cancel GPa and kN; a second to cancel mm² and m²; and a third to put the final answer in mm.

$$\delta = \frac{4 \cdot 30 \text{ kN} \cdot 80 \text{ cm}}{\pi (6 \text{ mm})^2 207 \text{ GPa}} \left| \frac{\text{GPa} \cdot \text{m}^2}{10^6 \text{ kN}} \right| \left| \frac{(10^3 \text{ mm})^2}{\text{m}^2} \right| \left| \frac{10 \text{ mm}}{\text{cm}} \right| = 4.1 \text{ mm}$$

Example #6: Weight of a Solid Bar in U.S. Units

Specific weight is a materials property, similar to density. It is the weight of an object divided by the object's volume, so $\gamma = \frac{W}{V}$. If we know the specific weight of a material and the volume of an object made of that material, we can find the weight of the object by rewriting the previous equation: $W = \gamma V$. The volume of a rod, pipe, or bar is the cross-sectional area times the length: $V = AL$. Calculate the weight of a 2 inch diameter, 3 foot long bar of steel. The specific weight of steel is 0.284 lb./in.³

Step 1 Combine the two equations to solve for weight: $W = \gamma AL$. Since the rod is round, the cross-sectional area is $A = \frac{\pi}{4} d^2$, therefore $W = \frac{\gamma \pi d^2 L}{4}$

Step 2 Draw a horizontal line and enter the numbers and units.

$$W = \frac{\gamma \pi d^2 L}{4} = \frac{0.284 \text{ lb.} \cdot \pi (2 \text{ in.})^2 3 \text{ ft.}}{\text{in.}^3 \cdot 4}$$

Step 3 The only unit conversion is feet to inches.

$$W = \frac{\gamma \pi d^2 L}{4} = \frac{0.284 \text{ lb.} \cdot \pi (2 \text{ in.})^2 3 \text{ ft.}}{\text{in.}^3 \cdot 4} \left| \frac{12 \text{ in.}}{\text{ft.}} \right. = 32.1 \text{ lb.}$$

Example #7: Weight of a Hollow Pipe in U.S. Units

Calculate the weight of a 3 foot long steel pipe having an outside diameter of 2.38 in. and an inside diameter of 2.07 in. The specific weight of steel is 0.284 lb./in.³

Step 1 From Example #6, $W = \gamma AL$. Since the pipe is hollow, the cross-sectional area is $A = \frac{\pi}{4} (d_o^2 - d_i^2)$, therefore $W = \frac{\gamma \pi (d_o^2 - d_i^2) L}{4}$

Step 2 Draw a horizontal line and enter the numbers and units.

$$W = \frac{\gamma \pi d^2 L}{4} = \frac{0.284 \text{ lb.} \cdot \pi [(2.38 \text{ in.})^2 - (2.07 \text{ in.})^2] 3 \text{ ft.}}{\text{in.}^3 \cdot 4}$$

Step 3 The only unit conversion is feet to inches.

$$= \frac{0.284 \text{ lb.} \cdot \pi [(2.38 \text{ in.})^2 - (2.07 \text{ in.})^2] 3 \text{ ft.}}{\text{in.}^3 \cdot 4} \left| \frac{12 \text{ in.}}{\text{ft.}} \right. = 0.92 \text{ lb.}$$

One common mistake in this type of problem is to solve $(d_o^2 - d_i^2)$ as $(d_o - d_i)^2$...it's not the same thing. While $[(2.38 \text{ in.})^2 - (2.07 \text{ in.})^2] = 1.38 \text{ in.}^2$, $[2.38 \text{ in.} - 2.07 \text{ in.}]^2 = 0.31 \text{ in.}^2$...a significant difference.

Example #8: Weight of a Solid Bar in SI Units

Calculate the weight of a 5 cm diameter, 2 meter long bar of steel, given the density of steel is 7.85 g/cm³

Step 1 Use $W = \frac{\gamma \pi d^2 L}{4}$ from Example #6. Specific weight is density times gravity: $\gamma = \rho g$, so $W = \frac{\rho g \pi d^2 L}{4}$.

Step 2 Draw a horizontal line and enter the numbers and units.

$$W = \frac{\rho g \pi d^2 L}{4} = \frac{7.85 \text{ g}}{\text{cm}^3} \frac{9.81 \text{ m}}{\text{s}^2} \frac{\pi (5 \text{ cm})^2 2 \text{ m}}{4}$$

Step 3 The SI unit of weight is the newton: $\text{N} = \frac{\text{kg m}}{\text{s}^2}$.

$$W = \frac{7.85 \text{ g}}{\text{cm}^3} \frac{9.81 \text{ m}}{\text{s}^2} \frac{\pi (5 \text{ cm})^2 2 \text{ m}}{4} \left| \frac{\text{kg}}{10^3 \text{ g}} \right| \left| \frac{10 \text{ cm}}{\text{m}} \right| \left| \frac{\text{N s}^2}{\text{kg m}} \right| = 60.5 \text{ N}$$

Notice the unit “g” for grams and the term “g” for gravity. In science and engineering, we tend to use roman type for units, and italic type for variables. Another example is a block sliding on an inclined plane, where “N” stands for newtons and “N” stands for normal force.