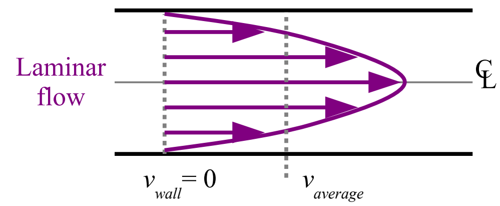
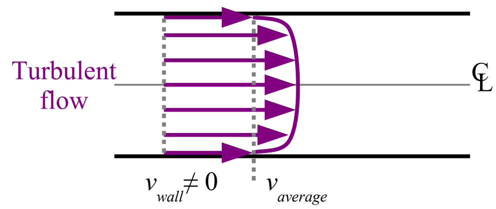
Fluid Flow & Bernoulli's Equation

Velocity Profiles



Previously we said that the velocity of hydraulic fluid in a pipe is the flow rate times the cross-sectional area of the pipe: $v=Q\ A$. Actually, this velocity is the average velocity of all the fluid molecules moving through the pipe. At low flow rates, all of the molecules move parallel to the axis of the pipe, and we have *laminar flow*. Molecules at the centerline move fastest, while fluid molecules at the wall remain attached to the wall. The velocity profile is parabolic.



At higher flow rates, fluid molecules do not follow straight paths; instead, eddies form in the flowstream, and we have *turbulent flow*. Molecules at the centerline move fastest, but the velocity profile is somewhat flattened. Notice that there is a velocity at the wall, so surface roughness affects the flow. In turbulent flow, the rougher the pipe wall, the greater the friction and pressure drop. Turbulence is undesirable in a hydraulic system because it increases the pressure drop in a pipe, so it is best to design hydraulic systems with laminar flow.

Introduction to Bernoulli's Equation

The 18^{th} century Swiss mathematician Daniel Bernoulli developed an equation for calculating pressures and velocities in a flowstream. Two hundred years later, his equation found practical application in the development of carburetors, propellors, and airplane wings.

In a hydraulic system, moving oil has kinetic energy, which is proportional to the square of the velocity of the oil. The pump adds energy to the hydraulic fluid by raising its pressure. Gravity can also add energy if the hydraulic lines drop in elevation. Energy is lost through friction in pipes; flow through valves, orifices, and fittings; motors; and elevation increases. All of these energy losses can be measured as a drop in pressure. As a mathematician, Bernoulli had little understanding of friction; he assumed that friction is negligible, and the energy in a fluid at one point of a hydraulic circuit equals the energy at a second point. If we include pumps, motors, and friction, we can modify Bernoulli's equation to say that the energy in a fluid at one point of a hydraulic system plus the energy added, minus the energy removed, equals the energy in a fluid at a second point.

The equation is $Z_1+\frac{p_1}{\gamma}+\frac{v_1^2}{2g}+H_P-H_M-H_L=Z_2+\frac{p_2}{\gamma}+\frac{v_2^2}{2g}$, where

Z	=	elevation change
p	=	pressure
γ	=	specific weight of the oil
v	=	velocity
g	=	acceleration of gravity
H_P	=	pump head
H_M	=	motor head
H_L	=	head loss due to friction in the lines

Let's look at the pieces of the equation, then put the pieces together to solve a practical problem.

Reynolds Number

We can characterize laminar and turbulent flow with a dimensionless number developed by Osborne Reynolds in the 19^{th} century. Reynolds number is the ratio of inertial forces to viscous forces; at low velocities, viscosity maintains a steady flow and we have laminar flow. At high velocities, inertia overcomes viscosity and we get turbulent flow.

In the textbook, the symbol for Reynolds number is N_R. In other textbooks you will see the symbol *Re* used instead.

Reynolds number is $N_R=\frac{v D \rho}{\mu}$ where v is the average fluid velocity, D is the inside pipe diameter, ρ is density, and μ is absolute viscosity. Since the kinematic viscosity $\nu=\frac{\mu}{\rho}$, $N_R=\frac{v D}{\nu}$. Be careful with this equation, because the Greek letter nu (ν) looks similar to the Roman letter vee (v). For hydraulic oil flowing through circular cross-section pipes, if

$N_R < 2000$ we have laminar flow.

The textbook provides four equations for Reynolds number: two in US Customary, two in SI, each set with either absolute viscosity or kinematic viscosity. These equations also use specific gravity instead of density.

The U.S. Customary equations are

$$N_R = \frac{7740\, v\, D\, SG}{\mu} \qquad N_R = \frac{7740\, v\, D}{\nu}$$

where

v	=	velocity (ft/s)
D	=	pipe diameter (in.)
SG	=	specific gravity of the oil
μ	=	absolute viscosity (cP)
ν	=	kinematic viscosity (cSt)

The SI equations are

$$N_R = \frac{1000\, v\, D\, SG}{\mu} \qquad N_R = \frac{1000\, v\, D}{\nu}$$

where

v	=	velocity (m/s)
D	=	pipe diameter (mm)
SG	=	specific gravity of the oil
μ	=	absolute viscosity (cP)
ν	=	kinematic viscosity (cSt)

The constants 7740 and 1000 include the unit conversions required to balance the equations. Units are provided in the examples below.

Example #1

A hydraulic pump delivers 1.5 gpm through a ½ in. diameter pipe. The oil has an absolute viscosity $\mu = 110\,\text{cP}$ and a specific gravity SG = 0.9. Do we have laminar flow or turbulent flow?

Step 1 Calculate the fluid velocity $v = \frac{Q}{A} = Q \frac{4}{\pi D^2} = \frac{1.5\,\text{gal.}}{\text{min.}} \frac{4}{\pi (0.5\,\text{in.})^2} \left| \frac{231\,\text{in.}^3}{\text{gal.}} \right| \frac{\text{min.}}{60\,\text{s}} \left| \frac{\text{ft.}}{12\,\text{in.}} \right. = 2.45\,\text{ft./s}$

Step 2 Calculate Reynolds number $N_R = \frac{7740\,\text{cP s}}{\text{ft. in.}} \frac{v\, D\, SG}{\mu} = \frac{7740\,\text{cP s}}{\text{ft. in.}} \frac{2.45\,\text{ft.}}{\text{s}} \frac{0.5\,\text{in.}\;\; 0.9}{110\,\text{cP}} = 77.6$

Since Reynolds number is less than 2000, flow is laminar.

Head Loss

Hydraulic circuits lose energy in several ways. The primary way is friction in pipes, which releases energy in the form of heat. We call this type of energy loss *head loss*. Head loss is $H_L = f \frac{L}{D} \frac{v^2}{2g}$ where f is the Darcy-Weisbach friction factor, L is the pipe length, D is the inside pipe diameter, v is the average fluid velocity, and g is the acceleration of gravity. In laminar flow through circular cross-section pipes, $f = \frac{64}{N_R}$.

Head loss can also occur across a filter; in this case, $H_L = \frac{\Delta p}{\gamma}$ where Δp is the pressure drop across the filter.

Example #2

Calculate the head loss in 2000 ft. of 1.5 in. diameter water pipe at a flow rate of 1 gpm. Water has a specific gravity of 1 and an absolute viscosity of 1.3 cP.

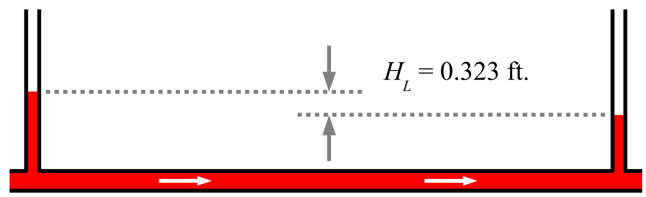
Step 1 Calculate the fluid velocity $v = \frac{Q}{A} = \frac{1\text{ gal.}}{\text{min.}} \frac{4}{\pi (1.5\text{ in.})^2} \left| \frac{231\text{ in.}^3}{\text{gal.}} \right| \frac{\text{min.}}{60\text{s}} \left| \frac{\text{ft.}}{12\text{ in.}} \right. = 0.182\text{ ft./s}$

Step 2 Calculate Reynolds number $N_R = \frac{7740\text{ cP s}}{\text{ft. in.}} \frac{0.182\text{ ft.}}{\text{s}} \frac{1.5\text{ in. } 1.0}{1.3\text{ cP}} = 1621$ Since N_R is less than 2000, flow is laminar.

Step 3 Calculate friction factor $f = \frac{64}{N_R} = \frac{64}{1621} = 0.0395$

Step 4 Calculate head loss $H_L = f \frac{L}{D} \frac{v^2}{2g} = 0.0395 \frac{2000\text{ ft.}}{1.5\text{ in.}} \frac{(0.182\text{ ft.})^2}{\text{s}^2} \frac{\text{s}^2}{2\ \ 32\text{ ft}} \left| \frac{12\text{ in.}}{\text{ft.}} \right. = 0.323\text{ ft.}$

Physically, you can measure head loss in a horizontal pipe with a pair of manometer pressure gauges. Install two T-fittings as shown, with transparent vertical pipes. If there is no flow, the levels will be the same. When fluid flows from left to right, the level in the righthand manometer tube is 0.323 ft. lower than the level in the lefthand manometer tube.

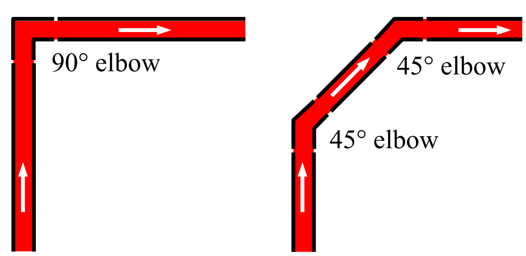


If the flow is turbulent, the the friction factor depends on Reynolds' number and the surface roughness of the pipe. Remember, in turbulent flow, the fluid velocity at the pipe wall is not zero…it's faster for a smooth pipe, slower for a rough pipe.

If you look on page 126 of the textbook, there's a table of surface roughness values for 7 different types of pipe material. Take these values with a grain of salt, because surface roughness can change over time, especially if the fluid is corrosive or deposits minerals, like water. When I worked as a co-op student at a water company, we would dig up 12" water mains that had a 3" effective inside diameter, due to mineral buildup over many decades. Cast iron pipes are replaced because of minerals, not because of corrosion.

In 1944, L.F. Moody plotted the friction factor of 21 different pipes for N_R = 4,000 to 100 million. You have a similar chart in the textbook, on page 127. All laminar flow occurs along this line on the left side of the chart. As the Reynolds Number increases, the friction factor drops. So you don't want the flow rate to be too slow, because friction is higher at very slow speeds. When you go from laminar to turbulent flow, the friction factor f increases by a factor of 4 to 8…this is a big deal because head loss is proportional to friction factor.

Head loss occurs in valves and fittings, usually because of a change in the flow direction or a change in the cross-sectional area that the fluid flows through. The head loss in a fitting $H_L = K \frac{v^2}{2g}$ where K is a constant for that fitting. Page 130 of the textbook lists K factors (a.k.a. loss coefficeints) for various fittings. We can use the K factor to design the plumbing in a circuit. For example, do you install one 90° elbow or two 45° elbows to make a 90° turn? The circuit with two 45° elbows looks smoother. From the table on page 130, the K value for a 90° bend is 0.75; for a 45° bend it is 0.42. Two 45° bends give us a total K value of 0.84, therefore the 90° bend is a better choice.



We can use the *equivalent length* technique to evaluate piping systems with valves and fittings. For a particular valve or fitting, the equivalent length $L_E = \frac{K D}{f}$. For example, if the equivalent length of a fitting is 25 feet, the fitting produces the same friction and pressure drop of 25 feet of straight pipe.

Example #3

Calculate the equivalent length of a ¼-open gate valve threaded onto a 1 in. diameter pipe carrying 30 gpm of oil with a kinematic viscosity of 100 cSt.

Step 1 Calculate the fluid velocity $v=\frac{Q}{A}=\frac{30\,\text{gal.}}{\text{min.}}\,\frac{4}{\pi(1\,\text{in.})^2}\left|\frac{231\,\text{in.}^3}{\text{gal.}}\right|\frac{\text{min.}}{60\,\text{s}}\left|\frac{\text{ft.}}{12\,\text{in.}}\right.=12.25\,\text{ft./s}$

Step 2 Calculate Reynolds number $N_R=\frac{7740\,\text{cSt s}}{\text{ft. in.}}\,\frac{12.25\,\text{ft.}}{\text{s}}\,\frac{1\,\text{in.}}{100\,\text{cSt}}=948$ Since N_R is less than 2000, flow is laminar.

Step 3 Calculate friction factor $f=\frac{64}{N_R}=\frac{64}{948}=0.0675$

Step 4 $K=24$ for a ¼-open gate valve with a 1" diameter pipe.

Calculate equivalent length $L_E=\frac{K\,D}{f}=\frac{24\;\;1\,\text{in.}}{0.0675}\left|\frac{\text{ft.}}{12\,\text{in.}}\right.=29.6\,\text{ft.}$

Using Bernoulli's Equation

Now let’s put everything together to solve for the pressures in different parts of a hydraulic system. An engineer needs to calculate these pressures in order to select the right pipe sizes and purchase pressure gauges in appropriate ranges.

We can use a 10-step process for solving Bernoulli's equation, $Z_1+\frac{p_1}{\gamma}+\frac{v_1^2}{2g}+H_P-H_M-H_L=Z_2+\frac{p_2}{\gamma}+\frac{v_2^2}{2g}$, where subscripts 1 and 2 refer to two different points in the hydraulic circuit.

Step 1 Draw the diagram & label pipe lengths, elevations, points of interest, directions of flow, etc.

Step 2 Write the Bernoulli equation & identify any terms that equal zero.

Step 3 Calculate fluid velocity from flow rate.

Step 4 Calculate Reynolds number N_R. If this number is less than 2000, then we have laminar flow, and we can use the remaining equations. Turbulent flow requires a different solution for the friction factor; you'll learn how to do it in MET 350.

Step 5 Calculate the friction factor *f*.

Step 6 Calculate the equivalent length of the fittings & valves.

Step 7 Calculate head loss due to friction in the pipes, fittings, valves, and strainers: H_L.

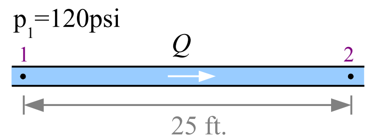
Step 8 Calculate pump head and motor head, H_P and H_M (if applicable).

Step 9 Calculate pressure due to the weight of a fluid in a tank (if applicable).

Step 10 Assemble Bernoulli’s equation from its parts, and solve.

Example #4

Oil flows at 7 gpm through a horizontal 1 inch ID pipe. Oil properties are $S.G.=0.9$ and $\nu=100\,\text{cSt}$. If the pressure is 120 psi at one point, what is the pressure 25 feet downstream?



Step 1 Draw the circuit. There are no pumps, motors, fittings, valves, or elevation changes.

Step 2 Terms that go to zero in the Bernoulli equation include elevation change (because $Z_1=Z_2$), velocity change (because $v_1=v_2$), pump head (because there is no pump between points 1 and 2), and motor head (because there is no motor between points 1 and 2).

$$\cancel{Z_1}+\frac{p_1}{\gamma}+\frac{\cancel{v_1^2}}{2g}+\cancel{H_P}-\cancel{H_M}-H_L=\cancel{Z_2}+\frac{p_2}{\gamma}+\frac{\cancel{v_2^2}}{2g}$$

$$\frac{p_1}{\gamma}-H_L=\frac{p_2}{\gamma}$$

Step 3 Flow rate is volume per unit time; velocity is distance per unit time. Divide flow rate by cross-sectional area to get velocity: $v=Q/A$. There is no leak of fluid between points 1 and 2, so the flow rate is the same at both points: $Q_1=Q_2$. Since the pipe diameter is constant, the velocity is the same at both points: $v_1=v_2$.

$$v_1=\frac{7\,\text{gal.}}{\text{min.}}\left|\frac{4}{\pi(1\,\text{in.})^2}\right|\frac{231\,\text{in.}^3}{\text{gal.}}\left|\frac{\text{min.}}{60\,\text{s}}\right|\frac{\text{ft.}}{12.\,\text{in.}}=2.860\,\text{ft./s}$$

Step 4 Calculate Reynolds number. Since $N_R<2000$ we have laminar flow.

$$N_R=\frac{7740\,v\,D}{\nu}=\frac{7740\,\text{cSt s}}{\text{ft. in.}}\,\frac{}{100\,\text{cSt}}\,\frac{2.860\,\text{ft.}}{\text{s}}\,\frac{1\,\text{in.}}{}=221.3$$

Step 5 Friction factor $f=\frac{64}{N_R}$.

$$f=\frac{64}{221.3}=0.289$$

Step 6 There are no fittings or valves, so the equivalent length of the system is the length of the pipe.

$$L=L_{pipe}=25\,\text{ft.}$$

Step 7 The hydraulic fluid loses some energy due to friction as it passes through the pipe. Head loss $H_L=f\frac{L}{D}\frac{v^2}{2g}$. Don't forget to square the velocity.

$$H_L=0.289\frac{25\,\text{ft.}}{1\,\text{in.}}\left|\frac{12\,\text{in.}}{\text{ft.}}\right|\frac{(2.860\,\text{ft./s})^2}{2(32.2\,\text{ft./s}^2)}=11.01\,\text{ft.}$$

Step 8 There is no pump or motor between points 1 and 2.

$$H_P=0,\quad H_M=0$$

Step 9 There is no tank, so there is no additional pressure to calculate.

Step 10 Bernoulli's equation for this problem is $\frac{p_1}{\gamma}-H_L=\frac{p_2}{\gamma}$ where γ is the specific weight of the oil, so $\gamma_{oil}=S.G._{oil}\gamma_{water}$. Solving Bernoulli's equation for pressure at point 2, $p_2=\left[\frac{p_1}{\gamma}-H_L\right]\gamma$. The equation is easier to solve as $p_2=p_1-\gamma H_L$ because fewer unit conversions are needed.

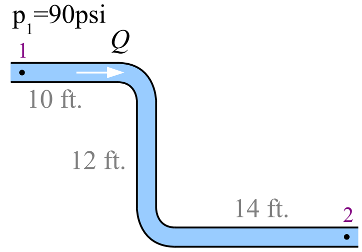
$$\gamma_{oil}=0.9\frac{62.4\,\text{lb.}}{\text{ft.}^3}=56.16\,\text{lb./ft.}^3$$

$$p_2=120\,\text{psi}-\frac{56.16\,\text{lb.}}{\text{ft.}^3}\,\frac{11.01\,\text{ft.}}{}\left|\frac{\text{ft.}^2}{(12\,\text{in.})^2}\right.=115.7\,\text{psi}$$

The pressure change between the two points is $p_2-p_1=-4.3\,\text{psi}$. The negative sign shows that the pressure has dropped from point 1 to point 2. In this problem, the pressure drop is due to friction in the pipe.

Example #5

Oil flows at 3 gpm through a horizontal 0.75 inch ID pipe for 10 feet, passes through a 90° standard elbow into a vertical pipe which drops for 12 feet, passes through a second 90° elbow into a horizontal pipe for another 14 feet. Oil properties are $\gamma = 54\,\text{lb./ft}^3$ and $\nu = 75\,\text{cSt}$. If the initial pressure is 90 psi, what is the final pressure?



Step 1 Draw the circuit. There are no pumps, motors, or valves, but we have two fittings and an elevation change.

Step 2 Terms that go to zero in the Bernoulli equation include velocity change (because $v_1 = v_2$), pump head (because there is no pump between points 1 and 2), and motor head (because there is no motor between points 1 and 2).

$$Z_1 + \frac{p_1}{\gamma} + \cancel{\frac{v_1^2}{2g}} + \cancel{H_P} - \cancel{H_M} - H_L = Z_2 + \frac{p_2}{\gamma} + \cancel{\frac{v_2^2}{2g}}$$

$$Z_1 + \frac{p_1}{\gamma} - H_L = Z_2 + \frac{p_2}{\gamma}$$

Step 3 Velocity $v = Q/A$ where Q is flow rate. There is no leak of fluid between points 1 and 2, so the flow rate is the same at both points: $Q_1 = Q_2$. Since the pipe diameter is constant, the velocity is the same at both points: $v_1 = v_2$.

$$v_1 = \frac{3\,\text{gal.}}{\text{min.}} \left| \frac{4}{\pi(1\,\text{in.})^2} \right| \frac{231\,\text{in.}^3}{\text{gal.}} \left| \frac{\text{min.}}{60\,\text{s}} \right| \frac{\text{ft.}}{12.\,\text{in.}} = 2.179\,\text{ft./s}$$

Step 4 Since $N_R < 2000$ we have laminar flow.

$$N_R = \frac{7740\,v\,D}{\nu} = \frac{7740\,\text{cSt s}}{\text{ft. in.}} \frac{}{75\,\text{cSt}} \frac{2.179\,\text{ft.}}{\text{s}} \frac{0.75\,\text{in.}}{} = 168.6$$

Step 5 Friction factor $f = \dfrac{64}{N_R}$.

$$f = \frac{64}{168.6} = 0.380$$

Step 6 The equivalent length of the system is the length of the pipe plus the equivalent length of the two elbows. Calculate the equivalent length of an elbow as $L_E = K\,D/f$. From the textbook, the loss coefficient of a 90° elbow is 0.75. We have two elbows, so *KD/f* is multiplied by 2.

$$L = L_{pipe} + \frac{K\,D}{f}$$

$$L = 10\,\text{ft.} + 12\,\text{ft.} + 14\,\text{ft.} + 2\left[\frac{0.75 \cdot 0.75\,\text{in}}{0.380} \left| \frac{\text{ft.}}{12\,\text{in.}} \right.\right] = 36.25\,\text{ft.}$$

Step 7 The hydraulic fluid loses some energy due to friction as it passes through the pipe and two elbows. Head loss $H_L = f\dfrac{L}{D}\dfrac{v^2}{2g}$.

$$H_L = 0.380 \frac{36.25\,\text{ft.}}{0.75\,\text{in.}} \left| \frac{12\,\text{in.}}{\text{ft.}} \right| \frac{(2.179\,\text{ft./s})^2}{2(32.2\,\text{ft./s}^2)} = 16.22\,\text{ft.}$$

Step 8 There is no pump or motor between points 1 and 2.

$$H_P = 0, \quad H_M = 0$$

Step 9 There is no tank, so there is no additional pressure to calculate.

Step 10 Solving Bernoulli's equation for pressure at point 2, $p_2 = \left[(Z_1 - Z_2) + \dfrac{p_1}{\gamma} - H_L\right]\gamma$. In this equation, Z_1 and Z_2 are the elevations at the two points. Pick one of these elevations to be zero, then use the system diagram to determine the other elevation. For example, if point 1 has elevation $Z_1 = 0$, then point 2 has elevation $Z_2 = -12\,\text{ft.}$, and the change in elevation $Z_1 - Z_2 = 0\,\text{ft.} - (-12\,\text{ft.}) = 12\,\text{ft.}$

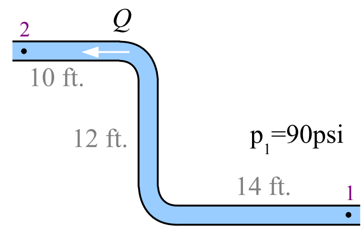
$$p_2 = \left[12\,\text{ft.} + \frac{90\,\text{lb.}}{\text{in.}^2} \frac{\text{ft.}^3}{54\,\text{lb.}} \left| \frac{(12\,\text{in.})^2}{\text{ft.}^2} \right. - 16.22\,\text{ft.}\right] \frac{54\,\text{lb.}}{\text{ft.}^3} \left| \frac{\text{ft.}^2}{(12\,\text{in.})^2} \right.$$

$$= 88.4\,\text{psi}$$

What if we had picked point 2 as the zero elevation? Then point 1 would have an elevation of +12 feet, and the change in elevation $Z_1 - Z_2 = 12\,\text{ft.} - 0\,\text{ft.} = 12\,\text{ft.}$ …the result is the same.

What if the flow direction were reversed? The math is the same, except $Z_1 - Z_2 = 0\text{ ft.} - 12\text{ ft.} = -12\text{ ft.}$

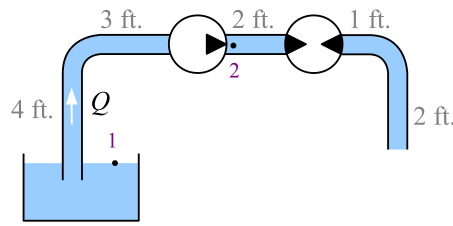
Now the pressure at point 2 is lower…there is a greater pressure drop because the oil is being pumped uphill.



$$p_2 = \left[-12\text{ ft.} + \frac{90\text{ lb.}}{\text{in.}^2}\frac{\text{ft.}^3}{54\text{ lb.}}\middle|\frac{(12\text{ in.})^2}{\text{ft.}^2} - 16.22\text{ ft.}\right]\frac{54\text{ lb.}}{\text{ft.}^3}\middle|\frac{\text{ft.}^2}{(12\text{ in.})^2}$$
$$= 79.4\text{ psi}$$

Now let's consider pumps and motors. Pumps add hydraulic power to a system, while motors extract hydraulic power from a system. The hydraulic power equation from Chapter 3 is the same for pumps and motors: $P = pQ$. Head pressure $p = \gamma_{oil} H$ and $S.G._{oil} = \gamma_{oil}/\gamma_{water}$. Substituting, pump head $H_P = \frac{p}{\gamma_{oil}} = \frac{P}{Q\gamma_{oil}} = \frac{P}{Q\ \ S.G._{oil}\gamma_{water}}$. With P in hp and Q in gpm, we can bake γ_{water} into a unit conversion constant, so $H_P = \frac{P}{Q\ \ S.G.}\frac{3950\text{ gpm ft.}}{\text{hp}}$. We use the same equation for motor head, $H_M = \frac{P}{Q\ \ S.G.}\frac{3950\text{ gpm ft.}}{\text{hp}}$.

Example #6



Oil flows at 8 gpm from a tank through 1 inch ID pipes and elbows, a pump, and a motor as shown. The pump adds 2 hp and the motor extracts 1 hp. Oil properties are $S.G.=0.9$ and $\nu=98\,\text{cSt}$. What is the pressure at point 2, immediately downstream of the pump?

Step 1 Draw the circuit. There are no motors or valves, but we have a pump, one fitting and an elevation change.

Step 2 Fluid velocity at the surface of the tank is effectively zero, because its surface area is hundreds of times the cross-sectional area of the pipe, and because all of the oil is returned to the tank, so the level remains constant.

The pressure at point 1 is zero. Since the motor is not between points 1 and 2, motor head is zero.

$$Z_1+\cancel{\frac{p_1}{\gamma}}+\cancel{\frac{v_1^2}{2g}}+H_P-\cancel{H_M}-H_L=Z_2+\frac{p_2}{\gamma}+\frac{v_2^2}{2g}$$

$$Z_1+H_p-H_L=Z_2+\frac{p_2}{\gamma}+\frac{v_2^2}{2g}$$

Step 3 Velocity $v_2=\frac{Q}{A}$.

$$v_2=\frac{8\,\text{gal.}}{\text{min.}}\left|\frac{4}{\pi(1\,\text{in.})^2}\right|\frac{231\,\text{in.}^3}{\text{gal.}}\left|\frac{\text{min.}}{60\,\text{s}}\right|\frac{\text{ft.}}{12.\,\text{in.}}=3.268\,\text{ft./s}$$

Step 4 Since $N_R<2000$ we have laminar flow.

$$N_R=\frac{7740\,v\,D}{\nu}=\frac{7740\,\text{cSt s}}{\text{ft. in.}}\,\frac{}{98\,\text{cSt}}\,\frac{3.268\,\text{ft.}}{\text{s}}\,\frac{1\,\text{in.}}{}=258.1$$

Step 5 Friction factor $f=\frac{64}{N_R}$.

$$f=\frac{64}{258.1}=0.248$$

Step 6 The equivalent length of the system is the length of the pipe plus the equivalent length of one elbow (the second elbow is not between points 1 and 2, so it is not included).

$$L=L_{pipe}+\frac{K\,D}{f}=4\,\text{ft.}+3\,\text{ft.}+\frac{0.75\cdot 1\,\text{in}}{0.248}\left|\frac{\text{ft.}}{12\,\text{in.}}\right.=7.252\,\text{ft.}$$

Step 7 The hydraulic fluid loses some energy due to friction as it passes through the pipe and elbow. Head loss $H_L=f\,\frac{L}{D}\,\frac{v^2}{2g}$. Since we'll need the term $\frac{v^2}{2g}$ later in Bernoulli's equation, let's calculate its value now.

$$\frac{v^2}{2g}=\frac{(3.286\,\text{ft./s})^2}{2(32.2\,\text{ft./s}^2)}=0.1658\,\text{ft.}$$

$$H_L=0.248\,\frac{7.252\,\text{ft.}}{1\,\text{in.}}\left|\frac{12\,\text{in.}}{\text{ft.}}\right|\frac{0.1658\,\text{ft.}}{}=3.58\,\text{ft.}$$

Step 8 The pump adds 2 hp as it pressurizes the hydraulic fluid.

$$H_P=\frac{2\,\text{hp}}{8\,\text{gpm}\;\;0.9}\left|\frac{3950\,\text{gpm ft.}}{\text{hp}}\right.=1097\,\text{ft.}$$

$$H_M=0$$

Step 9 Neither point 1 nor point 2 lie at the bottom of the tank, so there is no need to calculate the pressure at the bottom of the tank.

Step 10 Solving Bernoulli's equation for pressure at point 2, $p_2=\left[(Z_1-Z_2)+H_P-H_L-\frac{v_2^2}{2g}\right]\gamma$

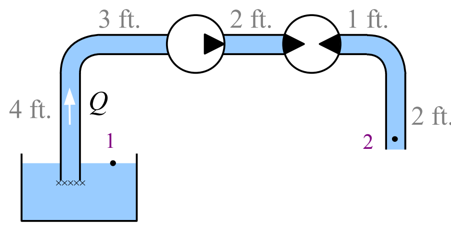
$$Z_1-Z_2=0\,\text{ft.}-4\,\text{ft.}=-4\,\text{ft.}$$

$$p_2=[-4\,\text{ft.}+1097\,\text{ft.}-3.58\,\text{ft.}-0.166\,\text{ft.}]\,\frac{0.9\;\;62.4\,\text{lb.}}{\text{ft.}^3}\left|\frac{\text{ft.}^2}{(12\,\text{in.})^2}\right.$$

$$=425\,\text{psi}$$

Example #7

Oil flows at 12 gpm from a tank through 1 inch ID pipes and elbows, and through a pump and motor as shown. The strainer at the inlet has a pressure drop of 2 psi. The pump adds 3 hp and the motor extracts 1 hp. Oil properties are $S.G.=0.9$ and $\nu=105\,\text{cSt}$. Calculate p_2.



Step 1 Draw the circuit. We have a pump, a motor, two fittings, and an elevation change.

Step 2 The pressure and velocity of the fluid at the surface of the tank are zero, as in the previous example.

$$Z_1+\frac{p_1}{\gamma}+\frac{v_1^2}{2g}+H_P-H_M-H_L=Z_2+\frac{p_2}{\gamma}+\frac{v_2^2}{2g}$$

$$Z_1+H_P-H_M-H_L=Z_2+\frac{p_2}{\gamma}+\frac{v_2^2}{2g}$$

Step 3 Velocity $v_2=\frac{Q}{A}$.

$$v_2=\frac{12\,\text{gal.}}{\text{min.}}\left|\frac{4}{\pi(1\,\text{in.})^2}\right|\frac{231\,\text{in.}^3}{\text{gal.}}\left|\frac{\text{min.}}{60\,\text{s}}\right|\frac{\text{ft.}}{12.\,\text{in.}}=4.902\,\text{ft./s}$$

Step 4 Since $N_R<2000$ we have laminar flow.

$$N_R=\frac{7740\,v\,D}{\nu}=\frac{7740\,\text{cSt s}}{\text{ft. in.}}\,\frac{}{105\,\text{cSt}}\,\frac{4.902\,\text{ft.}}{\text{s}}\,\frac{1\,\text{in.}}{}=361.3$$

Step 5 Friction factor $f=\frac{64}{N_R}$.

$$f=\frac{64}{361.3}=0.177$$

Step 6 The equivalent length of the system is the length of the pipe plus the equivalent length of both elbows.

$$L=4\,\text{ft.}+3\,\text{ft.}+2\,\text{ft.}+1\,\text{ft.}+2\,\text{ft.}+2\left[\frac{0.75\cdot 0.75\,\text{in}}{0.177}\left|\frac{\text{ft.}}{12\,\text{in.}}\right.\right]$$
$$=12.71\,\text{ft.}$$

Step 7 The hydraulic fluid loses some energy due to friction as it passes through the strainer, pipe, and two elbows.

The head loss due to the strainer is $\Delta p/\gamma$. Therefore, the total head loss is $H_L=f\frac{L}{D}\frac{v^2}{2g}+\frac{\Delta p}{\gamma}$.

$$\frac{v^2}{2g}=\frac{(4.902\,\text{ft./s})^2}{2(32.2\,\text{ft./s}^2)}=0.373\,\text{ft.}$$

$$H_L=0.177\frac{12.71\,\text{ft.}}{1\,\text{in.}}\left|\frac{12\,\text{in.}}{\text{ft.}}\right.\frac{0.373\,\text{ft.}}{}$$
$$+\frac{2\,\text{lb.}}{\text{in.}^2}\,\frac{\text{ft.}^3}{0.9\;\;62.4\,\text{lb.}}\left|\frac{(12\,\text{in.})^2}{\text{ft.}^2}\right.=15.20\,\text{ft.}$$

Step 8 The pump adds 3 hp as it pressurizes the hydraulic fluid, and the motor extracts 1 hp.

$$H_P=\frac{3\,\text{hp}}{12\,\text{gpm}\;\;0.9}\left|\frac{3950\,\text{gpm ft.}}{\text{hp}}\right.=1097\,\text{ft.}$$

$$H_M=\frac{1\,\text{hp}}{12\,\text{gpm}\;\;0.9}\left|\frac{3950\,\text{gpm ft.}}{\text{hp}}\right.=366\,\text{ft.}$$

Step 9 Neither point lies at the bottom of the tank, so there is no need to calculate pressure at the bottom of the tank.

Step 10 Solving Bernoulli's equation for pressure at point 2, $p_2=\left[(Z_1-Z_2)+H_P-H_M-H_L-\frac{v_2^2}{2g}\right]\gamma$

$$Z_1-Z_2=0\,\text{ft.}-2\,\text{ft.}=-2\,\text{ft.}$$

$$p_2=[-2\,\text{ft.}+1097\,\text{ft.}-366\,\text{ft.}-15.2\,\text{ft.}-0.373\,\text{ft.}]$$
$$\times\frac{0.9\;\;62.4\,\text{lb.}}{\text{ft.}^3}\left|\frac{\text{ft.}^2}{(12\,\text{in.})^2}\right.=278\,\text{psi}$$

Bernoulli's equation shows us where the energy is added to the system and where it is used or lost. In this problem, the pump adds 1097 ft. of head; all losses total 383 ft. of head. Elevation change consumes 0.5% of the 383 ft., the motor consumes 95.4%, friction consumes 4.0%, and the remaining 0.1% is used to move the fluid (kinetic energy).

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