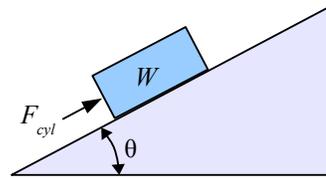


Inclined Plane Problems

The only prerequisite for MET 330 is algebra and trigonometry, although either Statics or Physics would help you with inclined plane problems. If you haven't taken either of these courses, then this handout will show you everything you need to know to solve this type of problem.

A block having a weight W is pushed up a ramp (the inclined plane) with a force F at a **constant speed**. This is an important point: if the speed of the block is changing, then we have a Dynamics problem, which requires more math. In this course, we are only dealing with Statics problems, where the block is either at rest or it is moving at a constant speed. A hydraulic cylinder operates at a constant speed during most of its stroke, because the pump provides a constant flow rate.



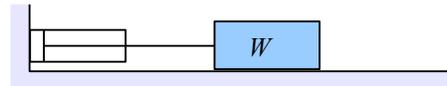
- F_{cyl} = cylinder force
- p = hydraulic pressure
- A = piston area
- d = piston diameter

In fluid power problems, the hydraulic cylinder provides force $F_{cyl} = pA = p \frac{\pi}{4} d^2$. If we calculate the force required to push the block up the slope, and we know the diameter, we can calculate the pressure by solving for p algebraically:

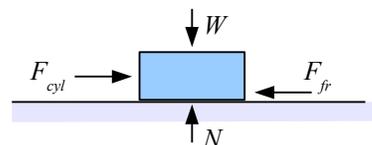
$p = \frac{F_{cyl}}{A} = \frac{4 F_{cyl}}{\pi d^2}$. If we calculate the force and know the hydraulic pressure, we can calculate the minimum required diameter for the cylinder to operate...so it becomes a design problem.

Block on a Horizontal Plane

The simplest case is a cylinder pushing a block that is sliding on a level, horizontal surface. Think of it as an inclined plane problem where the angle $\theta = 0$. There are four forces acting on the block:



1. Gravity acts downward on a block of mass m with a force $W = mg$, where g is the acceleration of gravity (9.81 m/s², or 32.2 ft./s²). We call this force "weight". The SI units of weight are the newton (N) or kilonewton (kN = 10³ N). The U.S. Customary unit of weight is the pound (lb.) Other U.S. Customary units of weight include the kip, or kilopound (1000 lb.) and the ton (2000 lb.).
2. If the block were not sitting on a solid surface, it would fall towards the center of the earth. The solid surface in this problem prevents the block from moving, so it must exert on a force on the block to support its weight. The force acts perpendicularly, or "normal" to the solid surface; we call the normal force N . Maybe this is an unfortunate choice of symbols; if you are solving an inclined plane problem in SI units, the italic capital letter N refers to a quantity (normal force) and the plain capital letter N refers to a unit (newton)...so you could have a confusing-looking equation like $N = 55 \text{ N}$.
3. The cylinder pushes on the block with a force F_{cyl} .



4. Friction between the block and the surface it rests on pushes in the opposite direction to F_{cyl} . This frictional force $F_{fr} = \mu N$, where μ is the coefficient of friction between the two surfaces.

If the block weighs 500 lb. and the friction coefficient is 0.2, what force is required to move the block?

First, draw a **Free-Body Diagram** of the block. This is a diagram which shows all of the forces acting on the isolated block.

Because we are solving a Statics problem (no accelerations), the sum of the forces in any given direction must equal zero. For example, in this problem the weight and the normal force are the only vertical forces. If we say “up” is positive, then

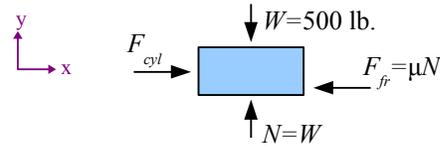
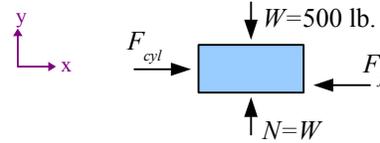
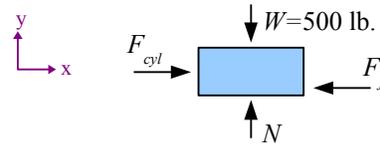
$$\sum F_y = 0 = N - W$$

$$N = W = 500 \text{ lb.}$$

The cylinder force and friction are the only horizontal forces. If we say “right” is positive, then

$$\sum F_x = 0 = F_{cyl} - F_{fr}$$

$$F_{cyl} = F_{fr} = \mu N = 0.2 \cdot 500 \text{ lb.} = 100 \text{ lb.}$$



Block Elevated at a 90° Angle

A cylinder can elevate a block vertically in the air at a constant speed. In this case, there is no friction between the block and a surface.

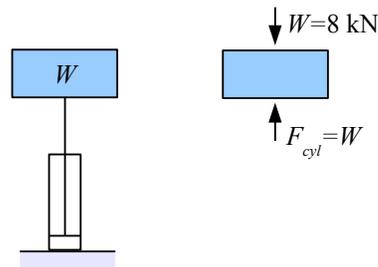
The free-body diagram has two forces in the vertical direction and none in the horizontal direction. If the block has a weight of 8 kN, then

$$\sum F_y = 0 = F_{cyl} - W$$

$$F_{cyl} = W = 8 \text{ kN}$$

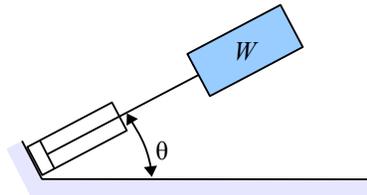
The force required to lift the block vertically equals the weight of the block.

Hydraulic jacks and hydraulic elevators are designed this way.

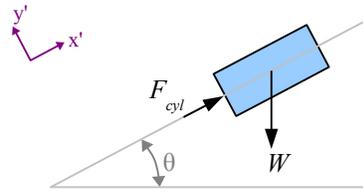


Block Elevated at Any Angle

A cylinder can elevate a block at any angle in the air at a constant speed. Again, there is no friction between the block and a surface.

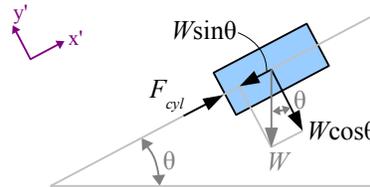


The free-body diagram of the block starts with the cylinder force at angle θ and the weight acting downwards.



These two forces do not act in the same direction, so we cannot use the Sum of the Forces to solve the problem as the diagram is currently drawn. Instead, we need to break the weight into two components: the portion parallel to the direction of the cylinder, and the portion acting perpendicular to the direction of the cylinder.

The portion acting parallel to the cylinder direction is $W \sin \theta$. The portion acting perpendicular to the cylinder direction is $W \cos \theta$.



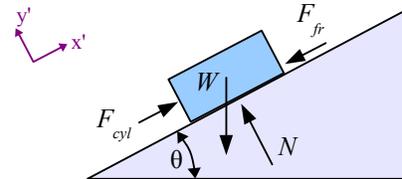
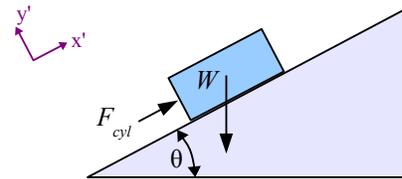
Along the cylinder direction, $\sum F_{x'} = 0 = F_{cyl} - W \sin \theta$. If we are raising a 1000 lb. block at an angle of 25° , the cylinder force is

$$\begin{aligned} F_{cyl} &= W \sin \theta \\ &= 1000 \text{ lb.} \sin 25^\circ \\ &= 423 \text{ lb.} \end{aligned}$$

Block Pushed Up an Inclined Plane

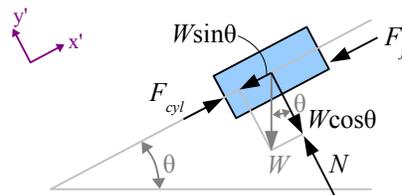
A cylinder pushes a 400 lb. block up a ramp at an angle $\theta = 27^\circ$. The friction coefficient between the block and the ramp is $\mu = 0.25$. There are four forces acting on the block:

1. The weight of the block, acting downwards.
2. The normal force exerted by the ramp, acting perpendicular to the ramp surface.
3. The cylinder force, acting parallel to the ramp surface, pushing the block up the ramp.
4. Friction, acting parallel to the ramp surface and opposite to the cylinder force.



Draw a free-body diagram with the four forces. Break the weight into two components, perpendicular and parallel to the ramp surface.

There are two forces acting perpendicular to the ramp surface: $\sum F_{y'} = 0 = N - W \cos \theta$, so the normal force is

$$\begin{aligned} N &= W \cos \theta \\ &= 400 \text{ lb.} \cos 27^\circ \\ &= 356 \text{ lb.} \end{aligned}$$


There are three forces acting parallel to the ramp surface. Let's call "up the ramp" positive, so the cylinder force is positive, while gravity and friction terms are negative:

$\sum F_x = 0 = F_{cyl} - W \sin \theta - F_{fr} = F_{cyl} - W \sin \theta - \mu N$, so the cylinder force is

$$\begin{aligned} F_{cyl} &= W \sin \theta + \mu N \\ &= 400 \text{ lb.} \sin 27^\circ + 0.25 (356 \text{ lb.}) \\ &= 182 \text{ lb.} + 89 \text{ lb.} \\ &= 271 \text{ lb.} \end{aligned}$$

The cylinder exerts 271 lb. to push the block up the ramp: 182 lb. to counteract gravity, and 89 lb. to counteract friction.

The cylinder force $F_{cyl} = p A = p \frac{\pi}{4} d^2$. If the hydraulic pressure is 250 psi, then the cylinder diameter

$$d = \sqrt{\frac{4 F_{cyl}}{\pi p}} = \sqrt{\frac{4 (271 \text{ lb.})}{\pi \cdot 250 \text{ lb.}}} = 2.08 \text{ in.}$$

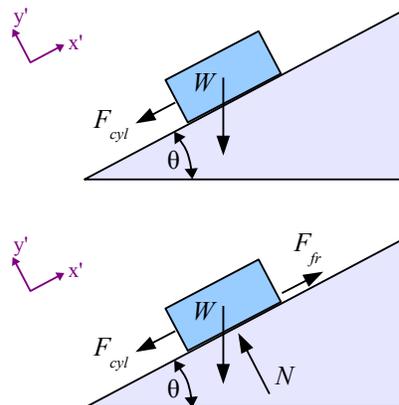
You can push a heavier block up the ramp in four ways:

1. Increase the cylinder diameter. Cylinder force increases with d^2 , so doubling the diameter will quadruple the force.
2. Increase the hydraulic pressure. Cylinder force increases proportionally to hydraulic pressure, so doubling the pressure will double the force.
3. Reduce the slope (but you will need a longer stroke and a longer ramp to raise the block to the same height).
4. Reduce the coefficient of friction by changing the surfaces of the block or ramp, or by greasing the ramp. In a manufacturing environment, ramps in production equipment are sometimes coated with Teflon strips to reduce friction.

Block Pulled Down an Inclined Plane

A cylinder pulls a 750 kg block down a ramp at an angle $\theta = 15^\circ$. The friction coefficient between the block and the ramp is $\mu = 0.30$. There are four forces acting on the block:

1. The weight of the block, acting downwards.
2. The normal force exerted by the ramp, acting perpendicular to the ramp surface.
3. The cylinder force, acting parallel to the ramp surface, pulling the block down the ramp.
4. Friction, acting parallel to the ramp surface and opposite to the cylinder force.



We know the mass of the block, 750 kg. Weight is mass times the acceleration of gravity, and the definition of the newton is $N = \frac{\text{kg m}}{\text{s}^2}$. Therefore, the weight of the block is:

$$\begin{aligned} W &= m g \\ &= \frac{1050 \text{ kg} \cdot 9.81 \frac{\text{m}}{\text{s}^2}}{\frac{\text{s}^2}{\text{kg m}} \left| \frac{\text{N s}^2}{\text{kg m}} \right| \frac{\text{kN}}{10^3 \text{ N}}} \\ &= 10.3 \text{ kN} \end{aligned}$$

Draw a free-body diagram with the four forces. Break the weight into two components, perpendicular and parallel to the ramp surface.

There are two forces acting perpendicular to the ramp surface: $\sum F_{y'} = 0 = N - W \cos \theta$, so the normal force is

$$\begin{aligned} N &= W \cos \theta \\ &= 10.3 \text{ kN} \cos 15^\circ \\ &= 9.95 \text{ kN} \end{aligned}$$

There are three forces acting parallel to the ramp surface: $\sum F_{x'} = 0 = -F_{cyl} - W \sin \theta + F_{fr} = -F_{cyl} - W \sin \theta + \mu N$. Pay close attention to the sign convention; in this problem, "up the ramp" is positive, so the cylinder force and gravity term are negative, and the friction term is positive. The cylinder force is

$$\begin{aligned} F_{cyl} &= -W \sin \theta + \mu N \\ &= -10.3 \text{ kN} \sin 15^\circ + 0.3(9.95 \text{ kN}) \\ &= -2.67 \text{ kN} + 2.98 \text{ kN} \\ &= 0.32 \text{ kN} \end{aligned}$$

The block pushes on the cylinder with a force of 2.67 kN due to its weight; the cylinder pulls on the block with a force of 2.98 kN to counteract friction; the net force exerted by the cylinder to pull the block down the slope is 0.32 kN.

