

**Equations for Chapters 16–20**

Note: the curly volume symbol in the book is not in a standard font; below,  $V$  = velocity and  $V$  = volume.

**Steady Heat Conduction, Convection, & Radiation in Walls, Cylinders, & Spheres**

$$\alpha = \frac{k}{\rho c_p}$$

$$\dot{Q}_{cond} = k A \frac{\Delta T}{L} = \frac{T_1 - T_2}{R_{wall}}$$

$$\dot{Q}_{conv} = h A (T_s - T_\infty) = \frac{(T_s - T_\infty)}{R_{conv}}$$

$$\dot{Q}_{emit} = \epsilon \sigma A_s T_s^4$$

$$\dot{Q}_{rad} = \epsilon \sigma A_s (T_s^4 - T_{surr}^4)$$

$$\dot{Q}_{total} = h_{combined} A_s (T_s - T_\infty)$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

$$\dot{Q}_{cond, cyl} = \frac{T_1 - T_2}{R_{cyl}}$$

$$\dot{Q}_{cond, sphere} = \frac{T_1 - T_2}{R_{sphere}}$$

$$\Delta T = \dot{Q} R$$

$$R_{wall} = \frac{L}{k A}$$

$$R_{cyl} = \frac{\ln(r_2/r_1)}{2 \pi L k}$$

$$R_{sphere} = \frac{r_2 - r_1}{4 \pi r_1 r_2 k}$$

$$R_{conv} = \frac{1}{h A_s}$$

$$R_{interface} = \frac{1}{h_c A} = \frac{R_c}{A}$$

$$R_{rad} = \frac{1}{h_{rad} A_s}$$

$$\text{Series: } R_{total} = R_1 + R_2 + R_3 + \dots$$

$$\text{Parallel: } \frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

$$h_{combined} = h_{conv} + h_{rad}$$

$$\dot{Q} = S k (T_1 - T_2)$$

$$W = m g$$

$$A_{circle} = \frac{\pi}{4} d^2$$

$$\text{Circumference of a circle} = \pi d$$

$$A_{sphere} = \pi d^2$$

$$V_{sphere} = \frac{\pi d^3}{6}$$

**Fins**

Very long fin:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-mx}$$

$$\dot{Q} = \sqrt{h p k A_c} (T_b - T_\infty)$$

$$m = \sqrt{\frac{h p}{k A_c}}$$

Adiabatic fin tip:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L-x)}{\cosh mL}$$

$$\dot{Q} = \sqrt{h p k A_c} (T_b - T_\infty) \tanh mL$$

$$L_c = L + \frac{A_c}{p}$$

$$\eta_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin, max}}$$

$$\epsilon_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no fin}}$$

**Lumped System Analysis**

$$\frac{T(x) - T_\infty}{T_i - T_\infty} = e^{-bt}$$

$$b = \frac{h A_s}{\rho c_p V} = \frac{h}{\rho c_p L_c}$$

$$Bi = \frac{h L_c}{k} < 0.1$$

$$L_c = \frac{V}{A_s}$$

$$\dot{Q} = h A_s [T(t) - T_i]$$

$$Q = m c_p [T(t) - T_i]$$

$$Q_{max} = m c_p [T_\infty - T_i]$$

**Transient Heat Conduction in Walls, Cylinders, Spheres, & Semi-infinite Solids**

$$\tau = \frac{\alpha t}{L^2} \text{ or } \frac{\alpha t}{r_o^2} \quad \theta_{wall} = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos\left(\frac{\lambda_1 x}{L}\right), \tau > 0.2$$

$$\theta_{cyl} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r/r_o), \tau > 0.2$$

$$\theta_{sphere} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r/r_o)}{(\lambda_1 r/r_o)}, \tau > 0.2$$

$$\text{Constant surface temperature} \quad \frac{T(x,t) - T_i}{T_s - T_i} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) \text{ and } \dot{q}_s(t) = \frac{k(T_s - T_i)}{\sqrt{\pi \alpha t}}$$

$$T_s = \text{constant}$$

$$\text{Constant heat flux} \quad T(x,t) - T_i = \frac{\dot{q}_s}{k} \left[ \sqrt{\frac{4\alpha t}{\pi}} \exp\left(\frac{-x^2}{4\alpha t}\right) \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) \right]$$

$$\dot{q}_s = \text{constant}$$

$$\text{Convection on the surface} \quad \frac{T(x,t) - T_i}{T_s - T_i} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right) \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)$$

$$\dot{q}_s(t) = h[T_\infty - T(0,t)]$$

$$\text{Energy pulse at the surface} \quad T(x,t) - T_i = \frac{e_s}{k\sqrt{\pi t/\alpha}} \exp\left(\frac{-x^2}{4\alpha t}\right)$$

$$e_s = \text{constant}$$

$$\text{Semi-infinite solids} \quad T_s = \frac{\sqrt{(k\rho c_p)_A} T_{A,i} + \sqrt{(k\rho c_p)_B} T_{B,i}}{\sqrt{(k\rho c_p)_A} + \sqrt{(k\rho c_p)_B}}$$

**External Forced Convection**

$$Nu = \frac{h L_c}{k} \quad Re = \frac{V L_c}{\nu} = \frac{\rho V L_c}{\mu}$$

$$\text{Flat plate, laminar } (Re_L < 5 \times 10^5): C_f = \frac{1.33}{Re_L^{1/2}} \text{ \& } Nu = 0.664 Re_L^{0.5} Pr^{1/3}$$

$$\text{Flat plate, laminar } (Re_L < 5 \times 10^5): Nu = 0.664 Re_L^{0.5} Pr^{1/3}$$

$$\text{Flat plate, turbulent } (5 \times 10^5 < Re_L < 10^7): Nu = 0.037 Re_L^{0.8} Pr^{1/3}$$

$$\text{Flat plate, laminar \& turbulent } (5 \times 10^5 < Re_L < 10^7): Nu = (0.037 Re_L^{0.8} - 871) Pr^{1/3}$$

**Internal Forced Convection**

$$\text{Constant surface heat flux: } \dot{Q} = \dot{m} c_p (T_e - T_i) \quad \dot{m} = \rho V A \quad D_h = 4 A_c / p \quad Re = \frac{V D}{\nu}$$

$$\text{Constant surface temperature: } \dot{Q} = h A_s \Delta T_{ln} = \dot{m} c_p (T_e - T_i) \text{ where } \Delta T_{ln} = \frac{T_i - T_e}{\ln[(T_s - T_e)/(T_s - T_i)]}$$

$$\text{Laminar flow in tubes: } f = 64 / Re$$

$$\text{Turbulent flow in tubes: } Nu = 0.125 f Re Pr^{1/3} \quad Nu = \frac{h D}{k}$$

$$\text{Turbulent flow in smooth tubes: } Nu = 0.023 Re^{0.8} Pr^{1/3}$$

**Natural Convection**

$$F_{\text{buoyancy}} = W_{\text{displaced fluid}}$$

$$\beta_{\text{ideal gas}} = \frac{1}{T}$$

$$Gr_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$$

$$\dot{Q} = h A_s (T_s - T_\infty)$$

$$Ra_L = Gr_L Pr$$

$$\text{Horizontal rectangular enclosures: } Nu = 1 + 1.44 \left[ 1 - \frac{1708}{Ra_L} \right]^+ + \left[ \frac{Ra_L^{1/3}}{18} - 1 \right]^+$$

$$\text{Inclined rectangular enclosures: } Nu = 1 + 1.44 \left[ 1 - \frac{1708}{Ra_L \cos \theta} \right]^+ \left( 1 - \frac{1708(\sin 1.8\theta)^{1.6}}{Ra_L \cos \theta} \right) + \left[ \frac{(Ra_L \cos \theta)^{1/3}}{18} - 1 \right]^+$$

Vertical rectangular enclosures:

$$Nu = 0.18 \left( \frac{Pr Ra_L}{0.2 + Pr} \right)^{0.29} \quad \text{for } 1 < \frac{H}{L} < 2$$

$$Nu = 0.22 \left( \frac{Pr Ra_L}{0.2 + Pr} \right)^{0.28} \left( \frac{H}{L} \right)^{-1/4} \quad \text{for } 2 < \frac{H}{L} < 10$$

$$\text{Concentric cylinders: } \dot{Q} = \frac{2\pi k_{\text{eff}}(T_i - T_o)}{\ln(D_o/D_i)} \quad \text{where } k_{\text{eff}} = 0.386k \left( \frac{Pr F_{\text{cyl}} Ra_L}{0.861 + Pr} \right)^{1/4} \quad \& \quad F_{\text{cyl}} = \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5}$$

$$\text{Conc. spheres: } \dot{Q} = \frac{\pi k_{\text{eff}} D_i D_o (T_i - T_o)}{L_c} \quad \text{where } k_{\text{eff}} = 0.74k \left( \frac{Pr F_{\text{sph}} Ra_L}{0.861 + Pr} \right)^{1/4} \quad \& \quad F_{\text{sph}} = \frac{L_c}{(D_i D_o)^4 (D_i^{-7/5} + D_o^{-7/5})^5}$$

**Metric Prefixes, Constants, and Unit Conversions**

n	= nano-	= $10^{-9}$
μ	= micro-	= $10^{-6}$
m	= milli-	= $10^{-3}$
c	= centi-	= $10^{-2}$
k	= kilo-	= $10^3$
M	= mega-	= $10^6$
G	= giga-	= $10^9$
T	= tera-	= $10^{12}$

$$\rho_{\text{water}} = 1000 \frac{\text{kg}}{\text{m}^3}$$

$$g = 9.81 \frac{\text{m}}{\text{s}^2}$$

$$P_0 = 1 \text{ atm} = 101 \text{ kPa}$$

$$\sigma = 5.6704 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$$

$$\text{Pa} = \frac{\text{N}}{\text{m}^2}$$

$$\text{N} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2}$$

$$\text{W} = \frac{\text{N} \cdot \text{m}}{\text{s}} = \frac{\text{J}}{\text{s}}$$

$$\text{J} = \text{N} \cdot \text{m} = \text{W} \cdot \text{s}$$

$$^\circ\text{C} + 273 = \text{K}$$