Equations for Chapters 16-22

Note: the curvy volume symbol in the book is not in a standard font; below, V = velocity and V = volume.

Steady Heat Conduction, Convection, & Radiation in Walls, Cylinders, & Spheres

$$\alpha = \frac{k}{\rho c_p}$$

$$\dot{Q}_{cond} = k A \frac{\Delta T}{L} = \frac{T_1 - T_2}{R_{wall}}$$

$$\dot{Q}_{conv} = h A (T_s - T_\infty) = \frac{(T_s - T_\infty)}{R_{conv}}$$

$$\dot{Q}_{emit} = \varepsilon \sigma A_s T_s^4$$

$$\dot{Q}_{rad} = \varepsilon \sigma A_s (T_s^4 - T_{surr}^4)$$

$$\dot{Q}_{total} = h_{combined} A_s (T_s - T_\infty)$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

$$\dot{Q}_{cond, cyl} = \frac{T_1 - T_2}{R_{cyl}}$$

$$\dot{Q}_{cond, sphere} = \frac{T_1 - T_2}{R_{sphere}}$$

$$\Delta T = \dot{Q} R$$

$$R_{wall} = \frac{L}{k A}$$

$$R_{cyl} = \frac{\ln(r_2 / r_1)}{2 \pi L k}$$

$$R_{sphere} = \frac{r_2 - r_1}{4 \pi r_1 r_2 k}$$

$$R_{conv} = \frac{1}{h A_s}$$

$$R_{interface} = \frac{1}{h_c A} = \frac{R_c}{A}$$

$$R_{rad} = \frac{1}{h_{rad} A_s}$$

Series: $R_{total} = R_1 + R_2 + R_3 + \ldots$

Parallel: $\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$

$$h_{combined} = h_{conv} + h_{rad}$$

$$\dot{Q} = S k (T_1 - T_2)$$

$$W = m g$$

$$A_{circle} = \frac{\pi}{4} d^2$$

Circumference of a circle $= \pi d$

$$A_{\text{sphere}} = \pi d^2$$

$$\mathsf{V}_{\text{sphere}} = \frac{\pi d^3}{6}$$

Fins

Very long fin:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-xm}$$

$$\dot{Q} = \sqrt{h p k A_c} (T_b - T_\infty)$$

$$m = \sqrt{\frac{h p}{k A_c}}$$

Adiabatic fin tip:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L - x)}{\cosh m L}$$

$$\dot{Q} = \sqrt{h p k A_c} (T_b - T_\infty) \tanh m L$$

$$L_c = L + \frac{A_c}{p}$$

$$\eta_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin, max.}}$$

$$\varepsilon_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\,fin}}$$

Lumped System Analysis

$$\frac{T(x) - T_\infty}{T_i - T_\infty} = e^{-bt}$$

$$b = \frac{h A_s}{\rho c_p \mathsf{V}} = \frac{h}{\rho c_p L_c}$$

$$\text{Bi} = \frac{h L_c}{k} < 0.1$$

$$L_c = \frac{\mathsf{V}}{A_s}$$

$$\dot{Q} = h A_s [T(t) - T_i]$$

$$Q = m c_p [T(t) - T_i]$$

$$Q_{max} = m c_p [T_\infty - T_i]$$

Transient Heat Conduction in Walls, Cylinders, Spheres, & Semi-infinite Solids

$\tau = \frac{\alpha t}{L^2}$ or $\frac{\alpha t}{r_o^2}$	$\theta_{wall} = \frac{T(x,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos\left(\frac{\lambda_1 x}{L}\right),\ \tau > 0.2$ $\theta_{cyl} = \frac{T(r,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r/r_o),\ \tau > 0.2$ $\theta_{sphere} = \frac{T(r,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r/r_o)}{(\lambda_1 r/r_o)},\ \tau > 0.2$
Constant surface temperature $T_s = \text{constant}$	$\frac{T(x,t)-T_i}{T_s - T_i} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)$ and $\dot{q}_s(t) = \frac{k(T_s - T_i)}{\sqrt{\pi \alpha t}}$
Constant heat flux $\dot{q}_s = \text{constant}$	$T(x,t) - T_i = \frac{\dot{q}_s}{k}\left[\sqrt{\frac{4\alpha t}{\pi}} \exp\left(\frac{-x^2}{4\alpha t}\right) \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)\right]$
Convection on the surface $\dot{q}_s(t) = h[T_\infty - T(0,t)]$	$\frac{T(x,t)-T_i}{T_s - T_i} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right) \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)$
Energy pulse at the surface $e_s = \text{constant}$	$T(x,t) - T_i = \frac{e_s}{k\sqrt{\pi t/\alpha}} \exp\left(\frac{-x^2}{4\alpha t}\right)$
Semi-infinite solids	$T_s = \frac{\sqrt{(k\rho c_p)_A} T_{A,i} + \sqrt{(k\rho c_p)_B} T_{B,i}}{\sqrt{(k\rho c_p)_A} + \sqrt{(k\rho c_p)_B}}$

External Forced Convection

$Nu = \frac{h L_c}{k}$ $\qquad$ $Re = \frac{V L_c}{\nu} = \frac{\rho V L_c}{\mu}$

Flat plate, laminar $(Re_L < 5 \times 10^5)$: $Nu = 0.664\, Re_L^{0.5}\, Pr^{1/3}$

Flat plate, turbulent $(5 \times 10^5 < Re_L < 10^7)$: $Nu = 0.037\, Re_L^{0.8}\, Pr^{1/3}$

Flat plate, laminar & turbulent $(5 \times 10^5 < Re_L < 10^7)$: $Nu = (0.037\, Re_L^{0.8} - 871)\, Pr^{1/3}$

Internal Forced Convection

Constant surface heat flux: $\dot{Q} = \dot{m} c_p (T_e - T_i)$ $\qquad$ $D_h = 4A_c/p$ $\qquad$ $f = 64/Re$

Constant surface temperature: $\dot{Q} = h A_s \Delta T_{\ln}$ where $\Delta T_{\ln} = \frac{T_i - T_e}{\ln[(T_s - T_e)/(T_s - T_i)]}$

Laminar flow in tubes: $f = 64/Re$

Turbulent flow in tubes: $Nu = 0.125\, f\, Re\, Pr^{1/3}$ $\qquad$ $Nu = \frac{hD}{k}$

Turbulent flow in smooth tubes: $Nu = 0.023\, Re^{0.8}\, Pr^{1/3}$

Natural Convection

$F_{buoyancy} = W_{displaced\ fluid}$

$\beta_{ideal\ gas} = \frac{1}{T}$

$Gr_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$

$\dot{Q} = h A_s (T_s - T_\infty)$

$Ra_L = Gr_L Pr$

Horizontal rectangular enclosures: $Nu = 1 + 1.44\left[1 - \frac{1708}{Ra_L}\right]^+ + \left[\frac{Ra_L^{1/3}}{18} - 1\right]^+$

Inclined rectangular enclosures: $Nu = 1 + 1.44\left[1 - \frac{1708}{Ra_L \cos\theta}\right]^+ \left(1 - \frac{1708(\sin 1.8\theta)^{1.6}}{Ra_L \cos\theta}\right) + \left[\frac{(Ra_L \cos\theta)^{1/3}}{18} - 1\right]^+$

Vertical rectangular enclosures:

$$Nu = 0.18\left(\frac{Pr\, Ra_L}{0.2 + Pr}\right)^{0.29} \quad \text{for } 1 < \frac{H}{L} < 2$$

$$Nu = 0.22\left(\frac{Pr\, Ra_L}{0.2 + Pr}\right)^{0.28}\left(\frac{H}{L}\right)^{-1/4} \quad \text{for } 2 < \frac{H}{L} < 10$$

Concentric cylinders: $\dot{Q} = \frac{2\pi k_{eff}(T_i - T_o)}{\ln(D_o/D_i)}$ where $k_{eff} = 0.386\text{k}\left(\frac{Pr\, F_{cyl}\, Ra_L}{0.861 + Pr}\right)^{1/4}$ & $F_{cyl} = \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5}$

Conc. spheres: $\dot{Q} = \frac{\pi k_{eff} D_i D_o (T_i - T_o)}{L_c}$ where $k_{eff} = 0.74\text{k}\left(\frac{Pr\, F_{sph}\, Ra_L}{0.861 + Pr}\right)^{1/4}$ & $F_{sph} = \frac{L_c}{(D_i D_o)^4 (D_i^{-7/5} + D_o^{-7/5})^5}$

Radiation

$E_b = \sigma T^4$

$f_{\lambda_1 - \lambda_2}(T) = f_{\lambda_2}(T) - f_{\lambda_1}(T)$

$\sum F_{ij} = 1$ in an enclosure

$A_i F_{i \to j} = A_j F_{j \to i}$

$F_{1 \to (2,3)} = F_{1 \to 2} + F_{1 \to 3}$

$\dot{Q}_{1 \to 2} = A_1 F_{1 \to 2} \sigma (T_1^4 - T_2^4)$

$$F_{1 \to 2} = \frac{(L_5 + L_6) - (L_3 + L_4)}{2 L_1} = \frac{\sum (\text{crossed strings}) - \sum (\text{uncrossed strings})}{2 \times \text{string on surface 1}}$$

Heat Exchangers

$$R=\frac{1}{h_i A_i}+\frac{R_{f,i}}{A_i}+\frac{\ln(D_o/D_i)}{2\pi k L}+\frac{R_{f,o}}{A_o}+\frac{1}{h_o A_o}=\frac{1}{U A_s}=\frac{1}{U_i A_i}=\frac{1}{U_o A_o}$$

$$\dot{Q}=\dot{m}\, h_{fg}$$

$$\dot{Q}=\dot{m}_c c_{pc}(T_{c,out}-T_{c,in})=\dot{m}_c c_{pc}(T_{h,in}-T_{h,out})$$

$$\dot{Q}=U A_s \Delta T_{lm}$$

$$\Delta T_{lm}=\frac{\Delta T_1-\Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

Parallel flow heat exchangers: $\Delta T_1=T_{h,in}-T_{c,in}$ and $\Delta T_2=T_{h,out}-T_{c,out}$

Counterflow heat exchangers: $\Delta T_1=T_{h,in}-T_{c,out}$ and $\Delta T_2=T_{h,out}-T_{c,in}$

Multipass heat exchangers: $\Delta T_{lm}=F\,\Delta T_{lm\,counterflow}$

Correction factor charts:

$$P=\frac{T_{tube\,outlet}-T_{tube\,inlet}}{T_{shell\,inlet}-T_{tube\,inlet}}$$

$$R=\frac{T_{shell\,inlet}-T_{shell\,outlet}}{T_{tube\,outlet}-T_{tube\,inlet}}$$

Metric Prefixes, Constants, and Unit Conversions

n	=	nano-	=	10^{-9}
μ	=	micro-	=	10^{-6}
m	=	milli-	=	10^{-3}
c	=	centi-	=	10^{-2}
k	=	kilo-	=	10^{3}
M	=	mega-	=	10^{6}
G	=	giga-	=	10^{9}
T	=	tera-	=	10^{12}

$\rho_{water}=1000\,\frac{\text{kg}}{\text{m}^3}$

$g=9.81\,\frac{\text{m}}{\text{s}^2}$

$P_0=1\,\text{atm}=101\,\text{kPa}$

$\sigma=5.6704\times10^{-8}\,\text{W}/\text{m}^2\text{K}^4$

$\text{Pa} = \frac{\text{N}}{\text{m}^2}$

$\text{N} = \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$

$\text{W} = \frac{\text{N}\cdot\text{m}}{\text{s}} = \frac{\text{J}}{\text{s}}$

$\text{J} = \text{N}\cdot\text{m} = \text{W}\cdot\text{s}$

$°\text{C}+273 = \text{K}$