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Applied Statistics and Probability for Engineers

Sixth Edition

Douglas C. Montgomery George C. Runger

Chapter 10
Statistical Inference for Two Samples

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10 Statistical Inference for Two Samples

CHAPTER OUTLINE

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Learning Objectives for Chapter 10

After careful study of this chapter, you should be able to do the following:

1. Structure comparative experiments involving two samples as hypothesis tests.
2. Test hypotheses and construct confidence intervals on the difference in means of two normal distributions.
3. Test hypotheses and construct confidence intervals on the ratio of the variances or standard deviations of two normal distributions.
4. Test hypotheses and construct confidence intervals on the difference in two population proportions.
5. Use the *P*-value approach for making decisions in hypothesis tests.
6. Compute power, Type II error probability, and make sample size decisions for two-sample tests on means, variances & proportions.
7. Explain & use the relationship between confidence intervals and hypothesis tests.

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10-1: Inference on the Difference in Means of Two Normal Distributions, Variances Known

Assumptions

1. Let $X_{11}, X_{12}, \ldots, X_{1n_1}$ be a random sample from population 1.
2. Let $X_{21}, X_{22}, \ldots, X_{2n_2}$ be a random sample from population 2.
3. The two populations X_1 and X_2 are independent.
4. Both X_1 and X_2 are normal.

$$E(\overline{X}_1 - \overline{X}_2) = E(\overline{X}_1) - E(\overline{X}_2) = \mu_1 - \mu_2$$

$$V(\overline{X}_1 - \overline{X}_2) = V(\overline{X}_1) + V(\overline{X}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

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10-1: Inference on the Difference in Means of Two Normal Distributions, Variances Known

The quantity

$$Z = \frac{\overline{X}_1 - \overline{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} \quad (10\text{-}1)$$

has a $N(0, 1)$ distribution.

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10-1.1 Hypothesis Tests on the Difference in Means, Variances Known

Null hypothesis: H_0: $\mu_1 - \mu_2 = \Delta_0$

Test statistic:

$$Z_0 = \frac{\overline{X}_1 - \overline{X}_2 - \Delta_0}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} \quad (10\text{-}2)$$

Alternative Hypotheses	*P*-Value	Rejection Criterion For Fixed-Level Tests
H_0: $\mu_1 - \mu_2 \neq \Delta_0$	Probability above $\|z_0\|$ and probability below $-\|z_0\|$, $P = 2[1 - \Phi(\|z_0\|)]$	$z_0 > z_{\alpha/2}$ or $z_0 < -z_{\alpha/2}$
H_1: $\mu_1 - \mu_2 > \Delta_0$	Probability above z_0, $P = 1 - \Phi(z_0)$	$z_0 > z_\alpha$
H_1: $\mu_1 - \mu_2 < \Delta_0$	Probability below z_0, $P = \Phi(z_0)$	$z_0 < -z_\alpha$

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EXAMPLE 10-1 Paint Drying Time

A product developer is interested in reducing the drying time of a primer paint. Two formulations of the paint are tested; formulation 1 is the standard chemistry, and formulation 2 has a new drying ingredient that should reduce the drying time. From experience, it is known that the standard deviation of drying time is 8 minutes, and this inherent variability should be unaffected by the addition of the new ingredient. Ten specimens are painted with formulation 1, and another 10 specimens are painted with formulation 2; the 20 specimens are painted in random order. The two sample average drying times are $\bar{x}_1 = 121$ minutes and $\bar{x}_2 = 112$ minutes, respectively. What conclusions can the product developer draw about the effectiveness of the new ingredient, using $\alpha = 0.05$?

The seven-step procedure is:

1. Parameter of interest: The difference in mean drying times, $\mu_1 - \mu_2$, and $\Delta_0 = 0$.

2. Null hypothesis: H_0: $\mu_1 - \mu_2 = 0$.

3. Alternative hypothesis: H_1: $\mu_1 > \mu_2$.

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EXAMPLE 10-1 Paint Drying Time - Continued

4. Test statistic: The test statistic is

$$z_0 = \frac{\bar{x}_1 - \bar{x}_2 - 0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

where $\sigma_1^2 = \sigma_2^2 = (8)^2 = 64$ and $n_1 = n_2 = 10$.

5. Reject H_0 if: Reject H_0: $\mu_1 = \mu_2$ if the P-value is less than 0.05.

6. Computations: Since $\bar{x}_1 = 121$ minutes and $\bar{x}_2 = 112$ minutes, the test statistic is

$$z_0 = \frac{121 - 112}{\sqrt{\frac{(8)^2}{10} + \frac{(8)^2}{10}}} = 2.52$$

7. **Conclusion:** Since z_0 = 2.52, from Table III $\Phi(2.52) = 0.994134$; the P-value is $P = 1 - \Phi(2.52) = 1 - 0.994134 = 0.0059$, so we reject H_0 at the $\alpha = 0.05$ level

Interpretation: We can conclude that adding the new ingredient to the paint significantly reduces the drying time.

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10-1.2 Type II Error and Choice of Sample Size

Use of Operating Characteristic Curves
(Appendix Charts VIIa, VIIb, VIIc, and VIId)

Two-sided alternative:

$$d = \frac{|\mu_1 - \mu_2 - \Delta_0|}{\sqrt{\sigma_1^2 + \sigma_2^2}} = \frac{|\Delta - \Delta_0|}{\sqrt{\sigma_1^2 + \sigma_2^2}}$$

One-sided alternative:

$$d = \frac{|\mu_1 - \mu_2 - \Delta_0|}{\sqrt{\sigma_1^2 + \sigma_2^2}} = \frac{|\Delta - \Delta_0|}{\sqrt{\sigma_1^2 + \sigma_2^2}}$$

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10-1.2 Type II Error and Choice of Sample Size

Sample Size Formulas

Two-sided alternative:

For the two-sided alternative hypothesis with significance level α, the sample size $n_1 = n_2 = n$ required to detect a true difference in means of Δ with power at least $1 - \beta$ is

$$n \simeq \frac{(z_{\alpha/2} + z_\beta)^2(\sigma_1^2 + \sigma_2^2)}{(\Delta - \Delta_0)^2} \qquad (10\text{-}5)$$

One-sided alternative:

For a one-sided alternative hypothesis with significance level α, the sample size $n_1 = n_2 = n$ required to detect a true difference in means of $\Delta(\neq\Delta_0)$ with power at least $1 - \beta$ is

$$n = \frac{(z_\alpha + z_\beta)^2(\sigma_1^2 + \sigma_2^2)}{(\Delta - \Delta_0)^2} \qquad (10\text{-}6)$$

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EXAMPLE 10-3 Paint Drying Time Sample Size

To illustrate the use of these sample size equations, consider the situation described in Example 10-1, and suppose that if the true difference in drying times is as much as 10 minutes, we want to detect this with probability at least 0.90. Under the null hypothesis, $\Delta_0 = 0$. We have a one-sided alternative hypothesis with $\Delta = 10$, $\alpha = 0.05$ (so $z_\alpha = z_{0.05} = 1.645$), and since the power is 0.9, $\beta = 0.10$ (so $z_\beta = z_{0.10} = 1.28$). Therefore, we may find the required sample size from Equation 10-6 as follows:

$$n = \frac{(z_\alpha + z_\beta)^2(\sigma_1^2 + \sigma_2^2)}{(\Delta - \Delta_0)^2}$$

$$= \frac{(1.645 + 1.28)^2[(8)^2 + (8)^2]}{(10 - 0)^2} = 11$$

This is exactly the same as the result obtained from using the OC curves.

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10-1.3 Confidence Interval on a Difference in Means, Variances Known

If $\bar{x}_1$ and $\bar{x}_2$ are the means of independent random samples of sizes n_1 and n_2 from two independent normal populations with known variance σ_1^2 and σ_2^2, respectively, a **100(1 −α)% confidence interval for $\mu_1 - \mu_2$** is

$$\bar{x}_1 - \bar{x}_2 - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \bar{x}_1 - \bar{x}_2 + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \quad (10\text{-}7)$$

where $z_{\alpha/2}$ is the upper $\alpha/2$ percentage point of the standard normal distribution

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EXAMPLE 10-4 Aluminum Tensile Strength

Tensile strength tests were performed on two different grades of aluminum spars used in manufacturing the wing of a commercial transport aircraft. From past experience with the spar manufacturing process and the testing procedure, the standard deviations of tensile strengths are assumed to be known. The data obtained are as follows: $n_1 = 10$, $\bar{x}_1 = 87.6$, $\sigma_1 = 1$, $n_2 = 12$, $\bar{x}_2 = 74.5$, and $\sigma_2 = 1.5$. If μ_1 and μ_2 denote the true mean tensile strengths for the two grades of spars, we may find a 90% confidence interval on the difference in mean strength $\mu_1 - \mu_2$ as follows:

$$\bar{x}_1 - \bar{x}_2 - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2$$

$$\le \bar{x}_1 - \bar{x}_2 + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$87.6 - 74.5 - 1.645\sqrt{\frac{(1)^2}{10} + \frac{(1.5)^2}{12}} \le \mu_1 - \mu_2$$

$$\le 87.6 - 74.5 + 1.645\sqrt{\frac{(1^2)}{10} + \frac{(1.5)^2}{12}}$$

$$12.22 \le \mu_1 - \mu_2 \le 13.98$$

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10-1: Inference for a Difference in Means of Two Normal Distributions, Variances Known

Choice of Sample Size

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 (\sigma_1^2 + \sigma_2^2) \qquad (10\text{-}8)$$

Remember to round up if n is not an integer.

This ensures that the level of confidence does not drop below 100(1 − α)%.

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10-1: Inference for a Difference in Means of Two Normal Distributions, Variances Known

One-Sided Confidence Bounds

Upper Confidence Bound

$$\mu_1 - \mu_2 \le \bar{x}_1 - \bar{x}_2 + z_\alpha \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \qquad (10\text{-}9)$$

Lower Confidence Bound

$$\bar{x}_1 - \bar{x}_2 - z_\alpha \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \qquad (10\text{-}10)$$

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10-2.1 Hypotheses Tests on the Difference in Means, Variances Unknown

Case 1: $\sigma_1^2 = \sigma_2^2 = \sigma^2$

We wish to test:

$$H_0: \mu_1 - \mu_2 = \Delta_0$$
$$H_1: \mu_1 - \mu_2 \neq \Delta_0$$

The **pooled estimator** of σ^2, denoted by S_p^2, is defined by

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} \qquad (10\text{-}12)$$

The quantity

$$T = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \qquad (10\text{-}13)$$

has a t distribution with $n_1 + n_2 - 2$ degrees of freedom.

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Tests on the Difference in Means of Two Normal Distributions, Variances Unknown and Equal

Null hypothesis: $H_0: \mu_1 - \mu_2 = \Delta_0$

Test statistic:

$$T_0 = \frac{\overline{X}_1 - \overline{X}_2 - \Delta_0}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \qquad (10\text{-}14)$$

Alternative Hypothesis	***P*-Value**	Rejection Criterion for Fixed-Level Tests
$H_1: \mu_1 - \mu_2 \neq \Delta_0$	Probability above $\lvert t_0\rvert$ and probability below $-\lvert t_0\rvert$	$t_0 > t_{\alpha/2, n_1+n_2-2}$ or $t_0 < -t_{\alpha/2, n_1+n_2-2}$
$H_1: \mu_1 - \mu_2 > \Delta_0$	Probability above t_0	$t_0 > t_{\alpha, n_1+n_2-2}$
$H_1: \mu_1 - \mu_2 < \Delta_0$	Probability below t_0	$t_0 < -t_{\alpha, n_1+n_2-2}$

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EXAMPLE 10-5 Yield from a Catalyst

Two catalysts are being analyzed to determine how they affect the mean yield of a chemical process. Specifically, catalyst 1 is currently in use, but catalyst 2 is acceptable. Since catalyst 2 is cheaper, it should be adopted, providing it does not change the process yield. A test is run in the pilot plant and results in the data shown in Table 10-1. Is there any difference between the mean yields? Use $\alpha = 0.05$, and assume equal variances.

Observation Number	Catalyst 1	Catalyst 2
1	91.50	89.19
2	94.18	90.95
3	92.18	90.46
4	95.39	93.21
5	91.79	97.19
6	89.07	97.04
7	94.72	91.07
8	89.21	92.75
	$\bar{x}_1 = 92.255$	$\bar{x}_2 = 92.733$
	$s_1 = 2.39$	$s_2 = 2.98$

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EXAMPLE 10-5 Yield from a Catalyst - Continued

The seven-step hypothesis-testing procedure is as follows:

1. **Parameter of interest:** The parameters of interest are μ_1 and μ_2, the mean process yield using catalysts 1 and 2, respectively.

2. **Null hypothesis:** H_0: $\mu_1 = \mu_2$

3. **Alternative hypothesis:** H_1: $\mu_1 \neq \mu_2$

4. **Test statistic:** The test statistic is

$$t_0 = \frac{\bar{x}_1 - \bar{x}_2 - 0}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

5. **Reject H_0 if:** Reject H_0 if the P-value is less than 0.05.

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EXAMPLE 10-5 Yield from a Catalyst - Continued

6. **Computations:** From Table 10-1 we have $\bar{x}_1 = 92.255$, $s_1 = 2.39$, $n_1 = 8$, $\bar{x}_2 = 92.733$, $s_2 = 2.98$, and $n_2 = 8$.

Therefore

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(7)(2.39)^2 + 7(2.98)^2}{8 + 8 - 2} = 7.30$$

$$s_p = \sqrt{7.30} = 2.70$$

and

$$t_0 = \frac{\bar{x}_1 - \bar{x}_2}{2.70\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{92.255 - 92.733}{2.70\sqrt{\frac{1}{8} + \frac{1}{8}}} = -0.35$$

7. **Conclusions:** From Appendix Table V we can find $t_{0.40,14} = 0.258$ and $t_{0.25,14} = 0.692$. Since, $0.258 < 0.35 < 0.692$, we conclude that lower and upper bounds on the P-value are $0.50 < P < 0.80$. Therefore, since the P-value exceeds $\alpha = 0.05$, the null hypothesis cannot be rejected.

Interpretation: At 5% level of significance, we do not have strong evidence to conclude that catalyst 2 results in a mean yield that differs from the mean yield when catalyst 1 is used.

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10-2.1 Hypotheses Tests on the Difference in Means, Variances Unknown

Case 2: $\sigma_1^2 \neq \sigma_2^2$

If H_0: $\mu_1 - \mu_2 = \Delta_0$ is true, the statistic

$$T_0^* = \frac{\bar{X}_1 - \bar{X}_2 - \Delta_0}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}} \qquad (10\text{-}15)$$

is distributed as t with degrees of freedom given by

$$v = \frac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\dfrac{\left(s_1^2/n_1\right)^2}{n_1 - 1} + \dfrac{\left(s_2^2/n_2\right)^2}{n_2 - 1}} \qquad (10\text{-}16)$$

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EXAMPLE 10-6 Arsenic in Drinking Water

Arsenic concentration in public drinking water supplies is a potential health risk. An article in the *Arizona Republic* (May 27, 2001) reported drinking water arsenic concentrations in parts per billion (ppb) for 10 metropolitan Phoenix communities and 10 communities in rural Arizona. The data follow:

Metro Phoenix ($\bar{x}_1 = 12.5$, $s_1 = 7.63$)	**Rural Arizona** ($\bar{x}_2 = 27.5$, $s_2 = 15.3$)
Phoenix, 3	Rimrock, 48
Chandler, 7	Goodyear, 44
Gilbert, 25	New River, 40
Glendale, 10	Apache Junction, 38
Mesa, 15	Buckeye, 33
Paradise Valley, 6	Nogales, 21
Peoria, 12	Black Canyon City, 20
Scottsdale, 25	Sedona, 12
Tempe, 15	Payson, 1
Sun City, 7	Casa Grande, 18

Determine if there is any difference in mean arsenic concentrations between metropolitan Phoenix communities and communities in rural Arizona.

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EXAMPLE 10-6 Arsenic in Drinking Water - Continued

The seven-step procedure is:

1. **Parameter of interest:** The parameters of interest are the mean arsenic concentrations for the two geographic regions, say, μ_1 and μ_2, and we are interested in determining whether $\mu_1 - \mu_2 = 0$.

2. **Non hypothesis:** H_0: $\mu_1 - \mu_2 = 0$, or H_0: $\mu_1 = \mu_2$

3. **Alternative hypothesis:** H_1: $\mu_1 \neq \mu_2$

4. **Test statistic:** The test statistic is $t_0^* = \dfrac{\bar{x}_1 - \bar{x}_2 - 0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$

5. **Reject H_0 if :** The degrees of freedom on are found as

$$v = \frac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\dfrac{(s_1^2/n_1)^2}{n_1 - 1} + \dfrac{(s_2^2/n_2)^2}{n_2 - 1}}$$

$$= \frac{\left[\dfrac{(7.63)^2}{10} + \dfrac{(15.3)^2}{10}\right]^2}{\dfrac{\left[(7.63)^2/10\right]^2}{9} + \dfrac{\left[(15.3)^2/10\right]^2}{9}} = 13.2 \simeq 13$$

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EXAMPLE 10-6 Arsenic in Drinking Water - Continued

Therefore, using $\alpha = 0.05$ and a fixed-significance-level test, we would reject H_0: $\mu_1 = \mu_2$ if $t_0^* > t_{0.025,13} = 2.160$ or if $t_0^* < -t_{0.025,13} = -2.160$.

6. **Computations:** Using the sample data we find

$$t_0^* = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \frac{12.5 - 27.5}{\sqrt{\dfrac{(7.63)^2}{10} + \dfrac{(15.3)^2}{10}}} = -2.77$$

7. **Conclusions:** Because $t_0^* = -2.77 < t_{0.025,13} = -2.160$, we reject the null hypothesis.

Interpretation: We can conclude that mean arsenic concentration in the drinking water in rural Arizona is different from the mean arsenic concentration in metropolitan Phoenix drinking water.

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10-2.2 Type II Error and Choice of Sample Size

Example 10-8 **Yield from Catalyst Sample Size**
Consider the catalyst experiment in Example 10-5. Suppose that, if catalyst 2 produces a mean yield that differs from the mean yield of catalyst 1 by 4.0%, we would like to reject the null hypothesis with probability at least 0.85. What sample size is required?

Using $s_p = 2.70$ as a rough estimate of the common standard deviation σ, we have $d = |\Delta|/2\sigma = |4.0|/[(2)(2.70)] = 0.74$. From Appendix Chart VII*e* with $d = 0.74$ and $\beta = 0.15$, we find $n^* = 20$, approximately. Therefore, because $n^* = 2n - 1$,

$$n = \frac{n^* + 1}{2} = \frac{20 + 1}{2} = 10.5 \simeq 11 \text{(say)}$$

and we would use sample sizes of $n_1 = n_2 = n = 11$.

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10-2.3 Confidence Interval on the Difference in Means, Variance Unknown

Case 1: $\sigma_1^2 = \sigma_2^2 = \sigma^2$

If $\bar{x}_1$, $\bar{x}_2$, s_1^2, and s_2^2 are the sample means and variances of two random samples of sizes n_1 and n_2, respectively, from two independent normal populations with unknown but equal variances, then a 100(1 - α)% confidence interval on the difference in means $\mu_1 - \mu_2$ is

$$\bar{x}_1 - \bar{x}_2 - t_{\alpha/2,\, n_1+n_2-2}\, s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_2 \le \bar{x}_1 - \bar{x}_2 + t_{\alpha/2,\, n_1+n_2-2}\, s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \qquad (10\text{-}19)$$

where $s_p = \sqrt{[(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2]/(n_1 + n_2 - 2)}$ is the pooled estimate of the common population standard deviation, and $t_{\alpha/2,\, n_1+n_2-2}$ is the upper α/2 percentage point of the *t* distribution with $n_1 + n_2 - 2$ degrees of freedom.

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Example 10-9 Cement Hydration

Ten samples of standard cement had an average weight percent calcium of $\bar{x}_1 = 90.0$ with a sample standard deviation of s_1 = 5.0, and 15 samples of the lead-doped cement had an average weight percent calcium of $\bar{x}_2 = 87.0$ with a sample standard deviation of s_2 = 4.0. Assume that weight percent calcium is normally distributed with same standard deviation. Find a 95% confidence interval on the difference in means, $\mu_1 - \mu_2$, for the two types of cement.

The pooled estimate of the common standard deviation is found as follows:

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = \frac{9(5.0)^2 + 14(4.0)^2}{10+15-2} = 19.52$$

$$s_p = \sqrt{19.52} = 4.4$$

The 95% confidence interval is found using Equation 10-19:

$$\bar{x}_1 - \bar{x}_2 - t_{0.025,23}\, s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \le \mu_1 - \mu_2 \le \bar{x}_1 - \bar{x}_2 + t_{0.025,23}\, s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

Upon substituting the sample values and using $t_{0.025,23}$ = 2.069,

$$90.0 - 87.0 - 2.069(4.4)\sqrt{\frac{1}{10}+\frac{1}{15}} \le \mu_1 - \mu_2 \le 90.0 - 87.0 + 2.069(4.4)\sqrt{\frac{1}{10}+\frac{1}{15}}$$

which reduces to $-0.72 \le m_1 - m_2 \le 6.72$

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10-2.3 Confidence Interval on the Difference in Means, Variance Unknown

Case 2: $\sigma_1^2 \neq \sigma_2^2$

If $\bar{x}_1$, $\bar{x}_2$, s_1^2, and s_2^2 are the means and variances of two random samples of sizes n_1 and n_2, respectively, from two independent normal populations with unknown and unequal variances, an approximate 100(1 - α)% confidence interval on the difference in means $\mu_1 - \mu_2$ is

$$\bar{x}_1 - \bar{x}_2 - t_{\alpha/2,v}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} \le \mu_1 - \mu_2 \le \bar{x}_1 - \bar{x}_2 + t_{\alpha/2,v}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} \qquad (10\text{-}20)$$

where v is given by Equation 10-16 and $t_{\alpha/2,v}$ the upper α/2 percentage point of the t distribution with v degrees of freedom.

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10-4: Paired *t*-Test

Null hypothesis: $H_0: \mu_D = \Delta_0$

Test statistic: $T_0 = \dfrac{\overline{D} - \Delta_0}{S_D / \sqrt{n}}$ **(10-24)**

Alternative Hypothesis	*P-Value*	Rejection Criterion for Fixed-Level Tests
$H_1: \mu_D \neq \Delta_0$	Probability above $\|t_0\|$ and probability below $-\|t_0\|$	$t_0 > t_{\alpha/2,n-1}$ *or* $t_0 < -t_{\alpha/2,n-1}$
$H_1: \mu_D > \Delta_0$	Probability above t_0	$t_0 > t_{\alpha,n-1}$
$H_1: \mu_D < \Delta_0$	Probability below t_0	$t_0 < -t_{\alpha,n-1}$

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Example 10-11 Shear Strength of Steel Girder

An article in the *Journal of Strain Analysis* [1983, Vol. 18(2)] reports a comparison of several methods for predicting the shear strength for steel plate girders. Data for two of these methods, the Karlsruhe and Lehigh procedures, when applied to nine specific girders, are shown in the table below.

Girder	Karlsruhe Method	Lehigh Method	Difference d_j
S1/1	1.186	1.061	0.125
S2/1	1.151	0.992	0.159
S3/1	1.322	1.063	0.259
S4/1	1.339	1.062	0.277
S5/1	1.2	1.065	0.135
S2/1	1.402	1.178	0.224
S2/2	1.365	1.037	0.328
S2/3	1.537	1.086	0.451
S2/4	1.559	1.052	0.507

Determine whether there is any difference (on the average) for the two methods.

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Example 10-11 Shear Strength of Steel Girder

The seven-step procedure is:

1. Parameter of interest: The parameter of interest is the difference in mean shear strength for the two methods.
2. Null hypothesis: H_0: $\mu_D = 0$
3. Alternative hypothesis: H_1: $\mu_D \neq 0$
4. Test statistic: The test statistic is $t_0 = \frac{\bar{d}}{s_d / \sqrt{n}}$
5. Reject H_0 if: Reject H_0 if the *P*-value is <0.05.
6. Computations: The sample average and standard deviation of the differences d_j are $\bar{d} = 0.2769$ and s_d = 0.1350, and so the test statistic is

$$t_0 = \frac{\bar{d}}{s_d/\sqrt{n}} = \frac{0.2769}{0.1350/\sqrt{9}} = 6.15$$

7. Conclusions: Because $t_{0.0005,8}$ = 5.041 and the value of the test statistic t_0 = 6.15 exceeds this value, the *P*-value is less than 2(0.0005) = 0.001. Therefore, we conclude that the strength prediction methods yield different results.
Interpretation: The data indicate that the Karlsruhe method produces, on the average, higher strength predictions than does the Lehigh method.

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A Confidence Interval for μ_D from Paired Samples

If $\bar{d}$ and s_D are the sample mean and standard deviation of the difference of *n* random pairs of normally distributed measurements, a 100(1 - α)% confidence interval on the difference in means 32 $\mu_D = \mu_1 - \mu_2$ is

$$\bar{d} - t_{\alpha/2,n-1} s_D/\sqrt{n} \le \mu_D \le \bar{d} + t_{\alpha/2,n-1} s_D/\sqrt{n} \quad (10\text{-}25)$$

where $t_{\alpha/2,n-1}$ is the upper α /2% point of the *t* distribution with *n* - 1 degrees of freedom.

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Example 10-12 Parallel Park Cars

The journal *Human Factors* (1962, pp. 375–380) reported a study in which n = 14 subjects were asked to parallel park two cars having very different wheel bases and turning radii. The time in seconds for each subject was recorded and is given in Table 10-4. From the column of observed differences, we calculate $\bar{d}$ = 1.21 and s_D = 12.68. Find the 90% confidence interval for $\mu_D = \mu_1 - \mu_2$.

$$\bar{d} - t_{0.05,13}\, s_D/\sqrt{n} \le \mu_D \le \bar{d} + t_{0.05,13}\, s_D/\sqrt{n}$$
$$1.21 - 1.771(12.68)/\sqrt{14} \le \mu_D \le 1.21 + 1.771(12.68)/\sqrt{14}$$
$$-4.79 \le \mu_D \le 7.21$$

Subject	1(x_{1j})	2(x_{2j})	(d_j)
1	37.0	17.8	19.2
2	25.8	20.2	5.6
3	16.2	16.8	-0.6
4	24.2	41.4	-17.2
5	22.0	21.4	0.6
6	33.4	38.4	-5.0
7	23.8	16.8	7.0
8	58.2	32.2	26.0
9	33.6	27.8	5.8
10	24.4	23.2	1.2
11	23.4	29.6	-6.2
12	21.2	20.6	0.6
13	36.2	32.2	4.0
14	29.8	53.8	-24.0

Table 10-4

Notice that the confidence interval on μ_D includes zero. This implies that, at the 90% level of confidence, the data do not support the claim that the two cars have different mean parking times μ_1 and μ_2. That is, the value $\mu_D = \mu_1 - \mu_2 = 0$ is not inconsistent with the observed data.

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10-5 Inferences on the Variances of Two Normal Populations

10-5.1 The *F* Distribution

We wish to test the hypotheses:

$$H_0: \sigma_1^2 = \sigma_2^2$$
$$H_1: \sigma_1^2 \ne \sigma_2^2$$

Let *W and Y* be independent chi-square random variables with u and v degrees of freedom respectively. Then the ratio

$$F = \frac{W/u}{Y/v} \qquad (10\text{-}28)$$

has the probability density function

$$f(x) = \frac{\Gamma\left(\frac{u+v}{2}\right)\left(\frac{u}{v}\right)^{u/2} x^{(u/2)-1}}{\Gamma\left(\frac{u}{2}\right)\Gamma\left(\frac{v}{2}\right)\left[\left(\frac{u}{v}\right)x + 1\right]^{(u+v)/2}}, \qquad 0 < x < \infty \qquad (10\text{-}29)$$

and is said to follow the distribution with u degrees of freedom in the numerator and v degrees of freedom in the denominator. It is usually abbreviated as $F_{u,v}$.

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10-5.2 Hypothesis Tests on the Ratio of Two Variances

Let $X_{11}, X_{12}, \ldots, X_{1n_1}$ be a random sample from a normal population with mean μ_1 and variance σ_1^2, and let $X_{21}, X_{22}, \ldots, X_{2n_2}$ be a random sample from a second normal population with mean μ_2 and variance σ_2^2. Assume that both normal populations are independent. Let S_1^2 and S_2^2 be the sample variances. Then the ratio

$$F = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2}$$

has an *F* distribution with $n_1 - 1$ numerator degrees of freedom and $n_2 - 1$ denominator degrees of freedom.

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10-5.2 Hypothesis Tests on the Ratio of Two Variances

Null hypothesis: $H_0: \sigma_1^2 = \sigma_2^2$

Test statistic: $$F_0 = \frac{S_1^2}{S_2^2} \qquad (10\text{-}31)$$

Alternative Hypotheses	Rejection Criterion
$H_1: \sigma_1^2 \neq \sigma_2^2$	$f_0 > f_{\alpha/2, n_1-1, n_2-1}$ or $f_0 < f_{1-\alpha/2, n_1-1, n_2-1}$
$H_1: \sigma_1^2 > \sigma_2^2$	$f_0 > f_{\alpha, n_1-1, n_2-1}$
$H_1: \sigma_1^2 < \sigma_2^2$	$f_0 < f_{1-\alpha, n_1-1, n_2-1}$

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Example 10-13 Semiconductor Etch Variability

Oxide layers on semiconductor wafers are etched in a mixture of gases to achieve the proper thickness. The variability in the thickness of these oxide layers is a critical characteristic of the wafer, and low variability is desirable for subsequent processing steps. Two different mixtures of gases are being studied to determine whether one is superior in reducing the variability of the oxide thickness. Sixteen wafers are etched in each gas. The sample standard deviations of oxide thickness are s_1 = 1.96 angstroms and s_2 = 2.13 angstroms, respectively. Is there any evidence to indicate that either gas is preferable? Use a fixed-level test with α = 0.05.

The seven-step hypothesis-testing procedure is:

1. Parameter of interest: The parameter of interest are the variances of oxide thickness σ_1^2 and σ_2^2.

2. Null hypothesis: $H_0: \sigma_1^2 = \sigma_2^2$

3. Alternative hypothesis: $H_1: \sigma_1^2 \neq \sigma_2^2$

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10-5 Inferences on the Variances of Two Normal Populations

4. Test statistic: The test statistic is $f_0 = \frac{s_1^2}{s_2^2}$

5. Reject H_0 if : Because $n_1 = n_2 = 16$ and α = 0.05, we will reject if $f_0 > f_{0.025,15,15} = 2.86$ or if $f_0 < f_{0.975,15,15} = 1/f_{0.025,15,15} = 1/2.86 = 0.35$.

6. Computations: Because $s_1^2 = (1.96)^2 = 3.84$ and $s_2^2 = (2.13)^2 = 4.54$, the test statistic is

$$f_0 = \frac{s_1^2}{s_2^2} = \frac{3.84}{4.54} = 0.85$$

7. Conclusions: Because $f_{0..975,15,15} = 0.35 < 0.85 < f_{0.025,15,15} = 2.86$, we cannot reject the null hypothesis $H_0: \sigma_1^2 = \sigma_2^2$ at the 0.05 level of significance.

__Interpretation:__ There is no strong evidence to indicate that either gas results in a smaller variance of oxide thickness.

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10-5.3 Type II Error and Choice of Sample Size

Appendix Charts VII*o*, VII*p*, VII*q*, and VII*r* provide operating characteristic curves for the *F*-test given in Section 10-5.1 for a = 0.05 and a = 0.01, assuming that $n_1 = n_2 = n$. Charts VII*o* and VII*p* are used with the two-sided alternate hypothesis. They plot b against the abscissa parameter

$$\lambda = \frac{\sigma_1}{\sigma_2} \qquad (10\text{-}30)$$

for various $n_1 = n_2 = n$. Charts VII*q* and VII*r* are used for the one-sided alternative hypotheses.

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Example 10-14 Semiconductor Etch Variability Sample Size

For the semiconductor wafer oxide etching problem in Example 10-13, suppose that one gas resulted in a standard deviation of oxide thickness that is half the standard deviation of oxide thickness of the other gas. If we wish to detect such a situation with probability at least 0.80, is the sample size $n_1 = n_2 = 20$ adequate ?

Note that if one standard deviation is half the other,

$$\lambda = \frac{\sigma_1}{\sigma_2} = 2$$

By referring to Appendix Chart VII*o* with $n_1 = n_2 = 20$ and $\lambda = 2$, we find that $\beta \simeq 0.20$. Therefore, if $\beta \simeq 0.20$, the power of the test (which is the probability that the difference in standard deviations will be detected by the test) is 0.80, and we conclude that the sample sizes $n_1 = n_2 = 20$ are adequate.

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10-5.4 Confidence Interval on the Ratio of Two Variances

If s_1^2 and s_2^2 are the sample variances of random samples of sizes n_1 and n_2, respectively, from two independent normal populations with unknown variances σ_1^2 and σ_2^2, then a 100(1 – α)% confidence interval on the ratio σ_1^2/σ_2^2 is

$$\frac{s_1^2}{s_2^2} f_{1-\alpha/2,\, n_2-1, n_1-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{s_1^2}{s_2^2} f_{\alpha/2,\, n_2-1, n_1-1} \qquad (10\text{-}33)$$

where $f_{\alpha/2, n_2-1, n_1-1}$ and $f_{1-\alpha/2, n_2-1, n_1-1}$ are the upper and lower $\alpha/2$ percentage points of the *F* distribution with n_2 – 1 numerator and n_1 – 1 denominator degrees of freedom, respectively.

.A confidence interval on the ratio of the standard deviations can be obtained by taking square roots in Equation 10-33.

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Example 10-15 Surface Finish for Titanium Alloy

A company manufactures impellers for use in jet-turbine engines. One of the operations involves grinding a particular surface finish on a titanium alloy component. Two different grinding processes can be used, and both processes can produce parts at identical mean surface roughness. The manufacturing engineer would like to select the process having the least variability in surface roughness. A random sample of n_1 = 11 parts from the first process results in a sample standard deviations s_1 =5.1 microinches, and a random sample of n_1 = 16 parts from the second process results in a sample standard deviation of s_2 = 4.7 microinches. Find a 90% confidence interval on the ratio of the two standard deviations, σ_1 / σ_2.

Assuming that the two processes are independent and that surface roughness is normally distributed, we can use Equation 10-33 as follows:

$$\frac{s_1^2}{s_2^2} f_{0.95,15,10} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{s_1^2}{s_2^2} f_{0.05,15,10}$$

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Example 10-15 Surface Finish for Titanium Alloy - Continued

$$\frac{(5.1)^2}{(4.7)^2}0.39 \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{(5.1)^2}{(4.7)^2}2.85$$

or upon completing the implied calculations and taking square roots,

$$0.678 \le \frac{\sigma_1}{\sigma_2} \le 1.832$$

Notice that we have used Equation 10-30 to find

$$f_{0.95,15,10} = 1/f_{0.05,10,15} = 1/2.54 = 0.39.$$

Interpretation: Since this confidence interval includes unity, we cannot claim that the standard deviations of surface roughness for the two processes are different at the 90% level of confidence.

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10-6.1 Large-Sample Test on the Difference in Population Proportions

We wish to test the hypotheses:

$$H_0: p_1 = p_2$$
$$H_1: p_1 \ne p_2$$

The following test statistic is distributed approximately as standard normal and is the basis of the test:

$$Z = \frac{\hat{P}_1 - \hat{P}_2 - (p_1 - p_2)}{\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}} \qquad (10\text{-}34)$$

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10-6.1 Large-Sample Test on the Difference in Population Proportions

Null hypothesis: H_0: $p_1 = p_2$

Test statistic:
$$Z_0 = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{\hat{P}(1-\hat{P})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \qquad (10\text{-}35)$$

Alternative Hypothesis	***P*-Value**	**Rejection Criterion for Fixed-Level Tests**
H_1: $p_1 \neq p_2$	Probability above $\lvert z_0 \rvert$ and probability below $-\lvert z_0 \rvert$. $P = 2[1 - \Phi(\lvert z_0 \rvert)]$	$z_0 > z_{\alpha/2}$ or $z_0 < -z_{\alpha/2}$
H_1: $p_1 > p_2$	Probability above z_0. $P = 1 - \Phi(z_0)$	$z_0 > z_\alpha$
H_1: $p_1 < p_2$	Probability below z_0. $P = \Phi(z_0)$	$z_0 < -z_\alpha$

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Example 10-16 St. John's Wort

Extracts of St. John's Wort are widely used to treat depression. An article in the April 18, 2001, issue of the *Journal of the American Medical Association* compared the efficacy of a standard extract of St. John's Wort with a placebo in 200 outpatients diagnosed with major depression. Patients were randomly assigned to two groups; one group received the St. John's Wort, and the other received the placebo. After eight weeks, 19 of the placebo-treated patients showed improvement, and 27 of those treated with St. John's Wort improved. Is there any reason to believe that St. John's Wort is effective in treating major depression? Use a =0.05.

The seven-step hypothesis testing procedure leads to the following results:

1. Parameter of interest: The parameters of interest are p_1 and p_2, the proportion of patients who improve following treatment with St. John's Wort (p_1) or the placebo (p_2).

2. Null hypothesis: H_0: $p_1 = p_2$

3. Alternative hypothesis: H_1: $p_1 \neq p_2$

4. Test Statistic: The test statistic is
$$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

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Example 10-16 St. John's Wort - Continued

where $\hat{p}_1 = 27/100 = 0.27$, $\hat{p}_2 = 19/100 = 0.19$, $n_1 = n_2 = 100$, and

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{19 + 27}{100 + 100} = 0.23$$

5. Reject H_0 if: Reject H_0: $p_1 = p_2$ if the P-value is less than 0.05.

6. Computations: The value of the test statistic is

$$z_0 = \frac{0.27 - 0.19}{\sqrt{0.23(0.77)\left(\frac{1}{100} + \frac{1}{100}\right)}} = 1.34$$

7. **Conclusions:** Since $z_0 = 1.34$, the P-value is $P = 2[1 - \Phi(1.34)] = 0.18$, we cannot reject the null hypothesis.

Interpretation: There is insufficient evidence to support the claim that St. John's Wort is effective in treating major depression.

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10-6.2 Type II Error and Choice of Sample Size

If the alternative hypothesis is two sided, the β-error is

$$\beta = \Phi\left[\frac{z_{\alpha/2}\sqrt{\bar{p}\bar{q}(1/n_1 + 1/n_2)} - (p_1 - p_2)}{\sigma_{\hat{P}_1 - \hat{P}_2}}\right] - \Phi\left[\frac{-z_{\alpha/2}\sqrt{\bar{p}\bar{q}(1/n_1 + 1/n_2)} - (p_1 - p_2)}{\sigma_{\hat{P}_1 - \hat{P}_2}}\right] \quad (10\text{-}37)$$

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10-6.2 Type II Error and Choice of Sample Size

If the alternative hypothesis is H_1: $p_1 > p_2$,

$$\beta = \Phi\left[\frac{z_\alpha \sqrt{\bar{p}\bar{q}(1/n_1 + 1/n_2)} - (p_1 - p_2)}{\sigma_{\hat{P}_1 - \hat{P}_2}}\right] \quad (10\text{-}38)$$

and if the alternative hypothesis is H_1: $p_1 < p_2$,

$$\beta = 1 - \Phi\left[\frac{-z_\alpha \sqrt{\bar{p}\bar{q}(1/n_1 + 1/n_2)} - (p_1 - p_2)}{\sigma_{\hat{P}_1 - \hat{P}_2}}\right] \quad (10\text{-}39)$$

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10-6.2 Type II Error and Choice of Sample Size

For the two-sided alternative, the common sample size is

$$n = \frac{[z_{\alpha/2}\sqrt{(p_1 + p_2)(q_1 + q_2)/2} + z_\beta \sqrt{p_1 q_1 + p_2 q_2}\,]^2}{(p_1 - p_2)^2} \quad (10\text{-}40)$$

where $q_1 = 1 - p_1$ and $q_2 = 1 - p_2$.

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10-6.3 Confidence Interval on the Difference in the Population Proportions

If $\hat{p}_1$ and $\hat{p}_2$ are the sample proportions of observations in two independent random samples of sizes n_1 and n_2 that belong to a class of interest, an **approximate two-sided 100(1 – α)% confidence interval on the difference in the true proportions $p_1 - p_2$ is**

$$\hat{p}_1 - \hat{p}_2 - z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$\leq p_1 - p_2 \leq \hat{p}_1 - \hat{p}_2 + z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} \quad \text{(10-41)}$$

where $z_{\alpha/2}$ is the upper α/2 percentage point of the standard normal distribution.

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Example 10-17 Defective Bearings

Consider the process of manufacturing crankshaft hearings described in Example 8-8. Suppose that a modification is made in the surface finishing process and that, subsequently, a second random sample of 85 bearings is obtained. The number of defective bearings in this second sample is 8. Therefore, because n_1 = 85, $\hat{p}_1 = 10/85 = 0.1176$, n_2 = 85, and $\hat{p}_2 = 8/85 = 0.0941$. Obtain an approximate 95% confidence interval on the difference in the proportion of detective bearings produced under the two processes.

$$\hat{p}_1 - \hat{p}_2 - z_{0.025}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$\leq p_1 - p_2 \leq \hat{p}_1 - \hat{p}_2 + z_{0.025}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

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Example 10-17 Defective Bearings - Continued

$$0.1176 - 0.0941 - 1.96\sqrt{\frac{0.1176(0.8824)}{85} + \frac{0.0941(0.9059)}{85}}$$
$$\leq p_1 - p_2 \leq 0.1176 - 0.0941 + 1.96\sqrt{\frac{0.1176(0.8824)}{85} + \frac{0.0941(0.9059)}{85}}$$

This simplifies to

$$-0.0685 \leq p_1 - p_2 \leq 0.1155$$

Interpretation: This confidence interval includes zero. Based on the sample data, it seems unlikely that the changes made in the surface finish process have reduced the proportion of defective crankshaft bearings being produced.

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10-7: Summary Table and Road Map for Inference Procedures for Two Samples

Table 10-5 Roadmap to Construct Confidence Intervals and Hypothesis Tests, Two-Sample Case

Function of the Parameters to be Bounded by the Confidence Interval or Tested with a Hypothesis	Symbol	Other Parameters?	Confidence Interval Section	Hypothesis Test Section	Comments
Difference in means from two normal distributions	$\mu_1 - \mu_2$	Standard deviations σ_1 and σ_2 known	10-1.3	10-1.1	
Difference in means from two arbitrary distributions with large sample sizes	$\mu_1 - \mu_2$	Sample sizes large enough that σ_1 and σ_2 are essentially known	10-1.3	10-1.1	Large sample size is often taken to be n_1 and $n_2 \geq 40$
Difference in means from two normal distributions	$\mu_1 - \mu_2$	Standard deviations σ_1 and σ_2 are unknown, and assumed equal	10-2.3	10-2.1	Case 1: $\sigma_1 = \sigma_2$
Difference in means from two symmetric distributions	$\mu_1 - \mu_2$			10-3	The Wilcoxon rank-sum test is a nonparametric procedure
Difference in means from two normal distributions	$\mu_1 - \mu_2$	Standard deviations σ_1 and σ_2 are unknown, and NOT assumed equal	10-2.3	10-2.1	Case 2: $\sigma_1 \neq \sigma_2$

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10-7: Summary Table and Road Map for Inference Procedures for Two Samples

Function of the Parameters to be Bounded by the Confidence Interval or Tested with a Hypothesis	Symbol	Other Parameters?	Confidence Interval Section	Hypothesis Test Section	Comments
Difference in means from two symmetric distributions	$\mu_1 - \mu_2$			10-3	The Wilcoxon rank-sum test is a nonparametric procedure
Difference in means from two normal distributions	$\mu_1 - \mu_2$	Standard deviations σ_1 and σ_2 are unknown, and NOT assumed equal	10-2.3	10-2.1	Case 2: $\sigma_1 \neq \sigma_2$
Difference in means from two normal distributions in a paired analysis	$\mu_D = \mu_1 - \mu_2$	Standard deviation of differences are unknown	10-4	10-4	Paired analysis calculates differences and uses a one-sample method for inference on the mean difference
Ratio of variances of two normal distributions	σ_1^2/σ_2^2	Means μ_1 and μ_2 unknown and estimated	10-5.4	10-5.2	
Difference in two population proportions	$p_1 - p_2$	None	10-6.3	10-6.1	Normal approximation to the binomial distribution used for the tests and confidence intervals

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Important Terms & Concepts of Chapter 10

Comparative experiments

Confidence intervals on:
- Differences
- Ratios

Critical region for a test statistic

Identifying cause and effect

Null and alternative hypotheses

1 & 2-sided alternative hypotheses

Operating Characteristic (OC) curves

Paired *t*-test

Pooled *t*-test

P-value

Reference distribution for a test statistic

Sample size determination for: Hypothesis tests
Confidence intervals

Statistical hypotheses

Test statistic

Wilcoxon rank-sum test

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