TECH 646 Analysis of Research in Industry and Technology PART IV

Ch. 17 Hypothesis Testing (2 of 2)

Lecture note based on the text book and supplemental materials:

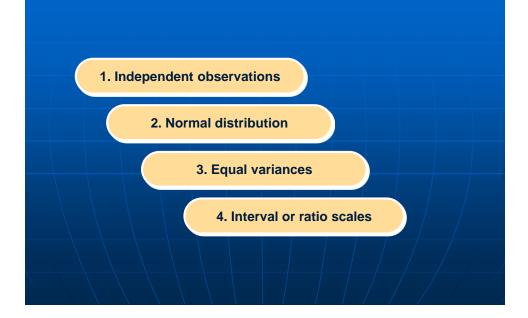
- Cooper, D.R., & Schindler, P.S., Business Research Methods (12th edition), McGraw-Hill/Irwin
- 2. D. C. Montgomery and G. C. Runger, Applied Statistics and Probability for Engineers, 6th Ed, Wiley

Paul I-Hai Lin, Professor <u>http://www.etcs.pfw.edu/~lin</u> A Core Course for M.S. iln Technology Program Purdue University Fort Wayne

Tests of Significance (Two general classes)

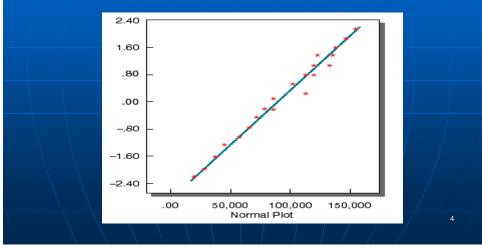
- Parametric Tests
 - More powerful
 - Data from Interval to Ratio Scale
- NonParametric Tests
 - Used to test hypotheses with data of Nominal and Ordinal Scale

Assumptions for Using Parametric Tests



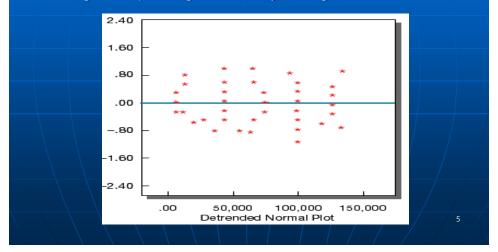
Probability Plots and Test of Normality

- Normal probability plot compares with a normal distribution
- The points fall within a narrow band along a straight line



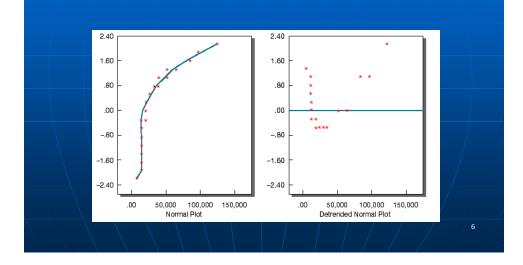
Probability Plots and Test of Normality

- Detrented plot the deviation from the straight line
- Expect the points to cluster without pattern around a straight line passing horizontally through 0.

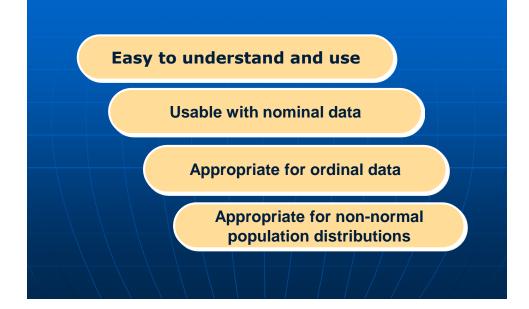


Probability Plots and Test of Normality

The variable is not normally distributed



Advantages of Nonparametric Tests



How to Select a Test

- How many samples are involved?
- If two or more samples, are involved, are the individual cases independent or related?
- Is the measurement scale nominal, ordinal, interval, or ratio?
- See Recommended Statistical Techniques next

Recommended Statistical Techniques by Measurement Level and Testing Situation

		Two-Sample Tests		k-Samp	le Tests
Measurement Scale	One-Sample Case	Related Samples	Independent Samples	Related Samples	Independent Samples
Nominal	 Binomial x² one-sample test 	• McNemar	 Fisher exact test x² two-samples test 	• Cochran Q	• x ² for k samples
Ordinal	 Kolmogorov- Smirnov one- sample test Runs test 	 Sign test Wilcoxon matched-pairs test 	 Median test Mann-Whitney U Kolmogorov- Smirnov Wald- Wolfowitz 	• Friedman two-way ANOVA	 Median extension Kruskal-Wallis one-way ANOVA
Interval and Ratio	 t-test Z test	 t-test for paired samples 	t-testZ test	 Repeated- measures ANOVA 	 One-way ANOVA n-way ANOVA

Questions Answered by One-Sample Tests

- Is there a difference between observed frequencies and the frequencies we would expect?
- Is there a difference between observed and expected proportions?
- Is there a significant difference between some measures of central tendency and the population parameter?



One-Sample: parametric Tests

- Determine the statistical significance between a sample distribution mean and a parameter
- Z Test

$$Z_0 = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

t-Test

$$T = \frac{\bar{X} - \mu_0}{S_{1/\sqrt{n}}}$$

When sample size n > 120, Z test and t-test are identical

One-Sample Parametric Tests

Real-world Application Examples

- Finding the average monthly balance of credit card holders compared to the average monthly balance five years ago.
- Comparing the failure rate of computers in a 20 hours test of quality specifications
- Discovering the proportion of people who would shop in a new district compared to the assumed population proportion.
- Comparing the average product revenues this year to last year's revenue

One-Sample t-Test Example

The hybrid-vehicle problem

- A sample of 100 vehicles
- Find the mean miles per gallon for the car is 52.5 mpg, with a standard deviation of 14
- Does these results indicate that population mean might still be 50?
 - 1. Null Hypothesis
 - H0 := 50 mpg
 - HA: > 50 mpg (one-tailed test)
 - 2. Statistical Test
 - t-Test is selected because the data are ratio measurement

 $T = \frac{\bar{X} - \mu_0}{S_{/\sqrt{n}}}$

One-Sample t-Test Example

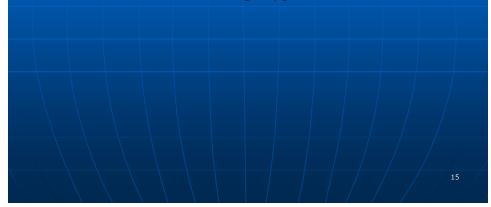
- Does these results indicate that population mean might still be 50?
 - 1. Null Hypothesis
 - H0 := 50 mpg
 - HA: > 50 mpg (one-tailed test)
 - 2. Statistical Test
 - t-Test is selected because the data are ratio measurement
 - 3. Significance level
 - $\alpha = 0.05, n = 100$
 - 4. Calculated value

 $T = \frac{\bar{x} - \mu_0}{s_{/\sqrt{n}}} = \frac{52.5 - 50}{14/\sqrt{100}} = \frac{2.5}{1.4} = 1.786, \quad d.f = n - 1 = 99$

- 5. Critical test value (Appendix D, Exhibit D-2, page 621)
 - Interpolated between d.f = 60 and d.f. = 120, A level of significance 0.05
 - Critical value of t = 1.66
- 6. Interpretation

One-Sample t-Test Example

- Does these results indicate that population mean might still be 50?
 - 6. Interpretation
 - The calculated value 1.786 > the critical value 1.66
 - Reject the null hypothesis
 - Conclude that the average mpg has increased



One-Sample t-Test Example - Summary

Null	Ho: = 50 mpg	
Statistical test	t <i>-test</i>	
Significance level	0.05, n=100	
Calculated value	1.786	
Critical test value	1.66	
	(from Appendix D, Exhibit D-2)	

One-Sample Chi-Square-Test Example

- A survey of student interest in Metro University Dinning Club.
- 200 students were interviewed about their interest in joining the club.
- The results are classified by living arrangement.
- Is there a significant difference among these students?



One Sample Chi-Square Test Example -Summary

Null	Ho: 0 = E	
Statistical test	One-sample chi-square	
Significance level	.05	
Calculated value	9.89	
Critical test value	7.82	

Chi-Square-Test Formula

The Ch-Square test formula

$$\chi^{2} = \sum_{i=1}^{k} \frac{(O_{i} - E_{i})^{2}}{E_{i}}$$

 O_i = observed number of cases categorized in the ith category

 E_i = expected number of cases in the ith category under H0

K = number of categories

d.f = k - 1

d.F = (r-1)(c-1); r - row, c - column

$$\chi^{2} = \frac{(16-27)^{2}}{27} + \frac{(13-12)^{2}}{12} + \frac{(16-12)^{2}}{12} + \frac{(15-9)^{2}}{9} = 4.48 + 0.08 + 1.33 + 4.0 = 9.89$$

Ch-SquareTest Example

- Is there a significant difference among these students?
 - 1. Null Hypothesis
 - H0 :Oi = Ei (The proportion in the population who intend to join the club is independent of living arrangement)
 - HA: Oi ≠ Ei
 - 2. Statistical Test
 - One sample Chi-square Test is selected because the response are classified into Nominal categories and there are sufficient observations
 - 3. Significance level
 - $\alpha = 0.05$
 - 4. Calculated value

$$\chi^{2} = \frac{(16-27)^{2}}{27} + \frac{(13-12)^{2}}{12} + \frac{(16-12)^{2}}{12} + \frac{(15-9)^{2}}{9} = 4.48 + 0.08 + 0$$

$$1.33 + 4.0 = 9.89$$

One Sample Chi-Square Test Example (Nominal data)

Living Arrangement	Intend to Join	Number Interviewed	Percent (no. interviewed/200)	Expected Frequencies (percent x 60)
Dorm/fraternity	16	90	45	27
Apartment/roomin g house, nearby	13	40	20	12
Apartment/roomin g house, distant	16	40	20	12
Live at home	15	30	15	9
Total	60	200	100	60

Chi-Square-Test

4. Calculated value: the Ch-Square test formula

$$\chi^{2} = \sum_{i=1}^{\kappa} \frac{(O_{i} - E_{i})^{2}}{E_{i}}$$

 O_i = observed number of cases categorized in the ith category

 E_i = expected number of cases in the ith category under H0

K = number of categories

d.f = k - 1

$$\chi^{2} = \frac{(16-27)^{2}}{27} + \frac{(13-12)^{2}}{12} + \frac{(16-12)^{2}}{12} + \frac{(15-9)^{2}}{9} = 4.48 + 0.08 + 1.33 + 4.0 = 9.89$$

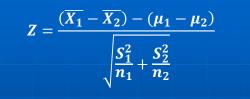
Ch-SquareTest Example

- Is there a significant difference among these students?
 - 5. Critical test value (Appendix D, Exhibit D-3, page 622)
 - d.f = 3
 - $\alpha = 0.05$
 - Critical value of 7.82
 - 6. Interpretation
 - The calculated value (9.89) is greater than the Critical value (7.82)
 - The null hypothesis is rejected.
 - We conclude that intending to join is dependent on living arrangement

One Sample Chi-Square Test Example

Living Arrangement	Intend to Join	Number Interviewed	Percent (no. interviewed/200)	Expected Frequencies (percent x 60)
Dorm/fraternity	16	90	45	27
Apartment/roomin g house, nearby	13	40	20	12
Apartment/roomin g house, distant	16	40	20	12
Live at home	15	30	15	9
Total	60	200	100	60

Two-Sample Parametric Tests

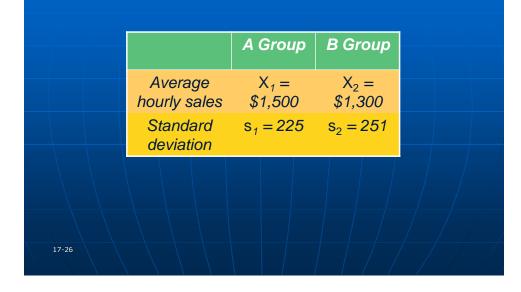


17-25

t

$$=\frac{(\overline{X_{1}}-\overline{X_{2}})-(\mu_{1}-\mu_{2})}{\sqrt{S_{p}^{2}(\frac{1}{n_{1}}+\frac{1}{n_{2}})}}$$





Two-Independent-Samples Tests Example

- Is there a significant difference among the group A and B?
 - 1. Null Hypothesis
 - H0 : There is no difference in sales results produced by two training methods
 - HA: Training method A produces sales results superior to those of method B
 - 2. Statistical Test
 - T-test: data are at least Interval, and the samples are independent
 - 3. Significance level
 - $\alpha = 0.05$ (one-tail test)
 - 4. The Calculated value
 - $t = 1.97, \ d.f = (11 1) + (11 1) = 20$
 - 5. Critical test value (Appendix D, Exhibit D-2, page 621)
 - Interpolated between d.f = 20, level of significance 0.05
 - Critical value of t = 1.725
 - 6. Interpretation
 - Since the calculated value is larger than the critical value (1.97 > 1.725), reject the null hypothesis and concluded that training method A is superior.

Two-Sample t-Test Example - summary

Null	Ho: A sales = B sales	
Statistical test	t <i>-test</i>	
Significance level	0.05 (one-tailed)	
Calculated value	1.97, d.f. = 20 = (11-1)+(11-1)	
Critical test value	1.725 (Appendix D, Exhibit D-2)	
	17-28	

SPSS Cross-Tab Procedure

- Yate's correction for continuity is applied for
 - Sample size n > 40, or
 - Sample is between 20 and 40 and the value of Ei are 5 or more
- The χ^2 of 2 x 2 table

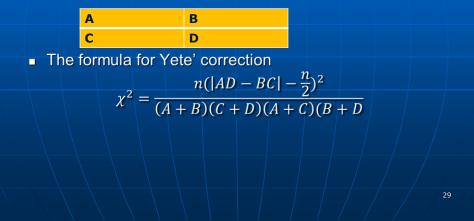
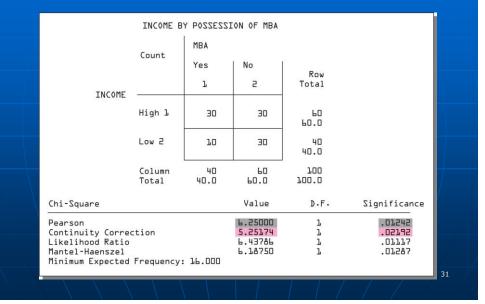


Exhibit 17-8 SPSS Cross-Tab Procedure (Income by Possession of MBA)

- Chi-square = 6.25, without the correction; with an observed level of significance of 0.01242
- Chi-square (with Yate's correction) = 5.25; significance level = 0.02192
 - If $\alpha = 0.01$; we would accept the null since 0.02192 > 0.01
- The literature is in conflict over the merits of Yates' correction
- Mantel-Haenszel test (used with Ordinal data)
- Likelihood ratio test (produce results similar to Pearson's Ch-square)

Exhibit 17-8 SPSS Cross-Tab Procedure (Income by Possession of MBA)



Two-Related Sample Tests

- Concern those situations in which persons, objects, or events are closely matched or the phenomena are measured twice.
- Parametric Test
 - T-test for independent samples is in appropriate here because of its assumption that observations are independent
- Non-Parametric Test
 - The McNemar test may be used (nominal or ordinal data)
 - Especially useful with before-after measurement of the same subjects.

Parametric Test

Parametric Test

- T-test for independent samples is in appropriate here because of its assumption that observations are independent
- The problem is solved by a formula in which the difference is found between each matched pair of observations.

$$t = \frac{\overline{D}}{S_D/\sqrt{n}} \qquad \overline{D} = \frac{\sum D}{n} S_D = \sqrt{\frac{\sum D - \frac{(\sum D)^2}{n}}{n-1}}$$

Exhibit 17-9 Sales Data for Pair-Samples t-Test (dollars in millions)

	Sales	Sales			
Company	Year 2	Year 1	Difference D	D^2	
GM	12693	123505	3427	11744329	
GE	54574	49662	4912	24127744	
Exxon	54574	78944	7712	59474944	
IBM	86656	59512	3192	10227204	
Ford	62710	92300	3846	14971716	
AT&T	96146	35173	939	881721	
Mobil	36112	48111	2109	4447881	
DuPont	50220	32427	2632	6927424	
Sears	35099	49975	3819	14584761	
Amoco	53794	20779	3187	10156969	
Total	23966		Σ <i>D</i> = 35781	Σ <i>D</i> = 157364693	
				·	

Pair-Samples t-Test

- Two years of Forbes sales data from 10 companies
- Null hypothesis
 - H0: μ = 0; there is no difference between year 1 and year 2 sales.
 - HA: $\mu \neq 0$; there is a difference between year 1 and year 2 sales.
- Statistical test:
 - The matched- or paired-samples t-test; measurement is ratio data
- Significance level
 - $\alpha = 0.01$, with n = 10 and d.f. = n-1
- Calculated value
- Critical test value
- Interpretation

Pair-Samples t-Test

- Calculated value: t = 6.27; d.f. = 9
- Critical test value: (Appendix D, Exhibit D-2, with d.f. = 9; two-tailed test, α = 0.01. The critical value is 3.25.
- Interpretation
 - The calculated value (6.27) > the critical value (3.25)
 - Reject the null hypothesis
 - Conclude there is a statistical significant difference between
 the two years sales

Paired-Samples t-Test

Null	Year 1 sales = Year 2 sales	
Statistical test	Paired sample t-test	
Significance level	0.01	
Calculated value	6.28, d.f. = 9	
Critical test value	3.25	
	(from Appendix D, Exhibit D-2)	
	3	

Exhibit 17-10 SPSS Output for 10 Companies Paired-Samples t-Test

The SPSS	output rou	unded 0.00	005 to 0.000

						1	
Variable	Number of Cases	Mean		andard /iation	Stand Err		
Year 2 Sales Year ⊥ Sales		62620.9 59038+8		777.649 372.871	10048 9836		
(Difference Mean)	Standard Deviation	Standard Error	Corr.	2-tail Prob.	t Value	Degrees of Freedom	2-tail Prob∙
3582.1000	1803.159	570.209	.999	.000	6.28	9	.000

SteelShelf Corp., New Concept Seating

- 200 employees were divided into equal groups reflecting their favorable or unfavorable view of the design using a questionnaire.
- After the campaign, the same 200 employees were asked again to complete the same questionnaire
- McNemar Test example (nominal or ordinal data)
- Test the significance of any observed change (before/after)

SteelShelf Corp.,New Concept Seating

- McNemar Test example (nominal or ordinal data)
- Test the significance of any observed change (before/after)
- A + D => total number of people changes
- B + C => total no-change response

Before	After Do Not Favor	After Favor	
Favor	А	В	
Do Not Favor	С	D	40

SteelShelf Corp.,New Concept Seating

McNemar Test example (nominal or ordinal data)

Before	After Do Not Favor	After Favor	
Favor	A=10	B=90	
Do Not Favor	C=60	D=40	
			41

SteelShelf Corp.,New Concept Seating The Hypothesis Testing Process

Null hypothesis

- H0: P(A) = P(D); new concept seating has no effect on employees' attitudes
- HA: P(A) ≠ P(D); new concept seating had effect on employees' attitudes

Statistical test:

The McNemar test; nominal data, before/after measurement of two related samples

Significance level

• α = 0.05, n = 200

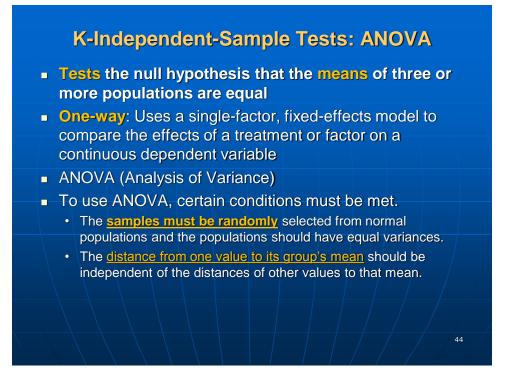
• Calculated value: $\chi^2 = 16.82$, d.f. = 1 (page 453)

 $\chi^2 = (|A - D| - 1)^2 / (A + D)$ with d.f. = 1

 $= (|10-40|-1)^2/(10+40) = 29^2/50 = 16.82$

SteelShelf Corp.,New Concept Seating The Hypothesis Testing Process

- Critical test value
 - Use Appendix D, Exhibit D-3
 - $\alpha = 0.05$, d.f. = 1 => find the critical value = 3.84
- Interpretation
 - The calculated value (16.82) > the critical value (3.83)
 - Reject null hypothesis
 - Conclude that the new concept had a significant positive effect on employees' attitudes.



K-Independent-Sample Tests: ANOVA

- In an ANOVA
 - Each group has its own mean and values that deviate from that mean.
 - The total deviation is the sum of the squared differences between each data point and the overall grand mean.
- Example: Airline Flight Service Rating
 - All data are hypothetical
 - Three airlines: 1 = Lufthansa; 2 = Malaysia Airlines; 3 = Cathay Pacific
 - Seat selection: 1 = economy; 2 = business
 - 20 passengers were randomly selected for each airlines (total of 60 passengers)
 - Concerns only the two columns: "Fight Service Rating 1" and Airline
 - The factor, airline, is the grouping variable for three carriers

K-Independent-Sample Tests: ANOVA

- The test statistics for ANOVA is F ratio
- F = between-groups variance/within-groups variance

```
= Mean square between/Mean square within
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where

Mean square _{between} = SumOfSquares _{between}/d.f. _{between}

Mean square $_{within}$ = SumOfSquares $_{within}$ /d.f. $_{within}$

3-Airline Service Rating Testing Procedure

Null hypothesis

- H0 : μ A1 = μ A2 = μ A3
- HA: $\mu A1 \neq \mu A2 \neq \mu A3$ (The means are not the same)

Statistical test

- The F test is chosen
- K independent samples, Interval data
- Accept the assumptions of ANOVA
- Significance level
 - $\alpha = 0.05$, d.f. =[numerator (k-1) = (3-1) = 2],
 - [denominator (n-k) = (60-3) = 57] = (2, 57)
- Calculated value
 - F = MSb/MSw = 5822.017/205.695 = 28.304

Exhibit 17-12 Three-Airlines: ANOVA Example

	Model Summary							
Source	d.f.	Sum of Squares	Mean Square	F Value	p Value			
Model (airline)	2	11644.033	5822.017	28.304	0.0001			
Residual (error)	57	11724.550	205.694					
Total	59	23368.583						

		Means	Means Table					
		Count	Mean	Std. Dev.	Std. Error			
	Lufthansa	20	38.950	14.006	3.132			
	Malaysia Airlines	20	58.900	15.089	3.374			
	Cathay Pacific	20	72.900	13.902	3.108			
A	ll data are hypoth	netical						
				$\left \right /$	48			

3-Airline Service Rating Testing Procedure

Critical test value

- Use Exhibit D-8 (pp. 626), with d.f. = (2, 57)
- Critical test value = 3.16

Interpretation

- Calculated value (28.304) > Critical value (3.16)
- · Reject the null hypothesis
- P value = 0.0001 < alpha (0.05) => a second method to reject Null
- Conclude that there is a significant difference in flight service ratings

Scheffe's S Multiple Comparison (Range Test) Procedure

- With an ANOVA, we cannot tell which pairs are not equal. We can use a post hoc test to determine where the differences lie.
- pp. 454-456

	Verses	Diff	Crit. Diff.	p Value	
Lufthansa	Malaysia Airlines	19,950	11.400	.0002	
	Cathay Pacific	33.950	11.400	.0001	
Malaysia Airlines	Cathay Pacific	14.000	11.400	.0122	

l	menubi	e com			uuies	
	Complex Comparis ons	Pairwise Comparis ons	Equal <i>n</i> 's Only	Unequal <i>n</i> 's	Equal Variances Assumed	Unequal Variances Not Assumed
Fisher LSD	Х			Х	Х	
Bonferroni	Х		Х	Х		
Tukey HSD	Х		Х		Х	
Tukey- Kramer	Х			Х	Х	
Games- Howell	Х			Х		Х
Tamhane T2	Х			Х		Х
Scheffé S		Х	Х	Х	Х	
Brown- Forsythe		Х	Х	Х		Х
Newman- Keuls	Х				Х	
Duncan	Х				Х	
Dunnet's T3						Х
Dunnet's C						Х

Multiple Comparison Procedures

Multiple Comparison Procedures

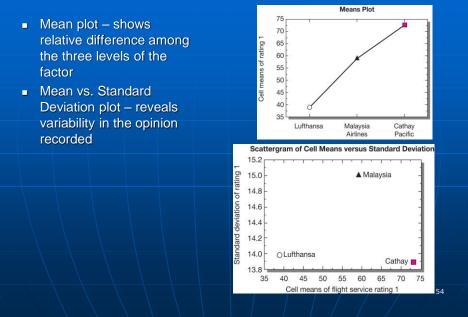
•	Fisher LSD (Least Significant Difference), http://support.minitab.com/en-us/minitab/17/topic-
	library/modeling-statistics/anova/multiple-comparisons/what-is-
	fisher-s-lsd-method/
	Bonferroni,
	http://www.itl.nist.gov/div898/handbook//prc/section4/prc473.ht
	<u>m</u>
	Turkey HSD (Honest Significant Test),
	https://en.wikipedia.org/wiki/Tukey's_range_test
_	Turkey-Krammer (work with unegal sample sizes),
	http://www.itl.nist.gov/div898/handbook/prc/section4/prc471.ht
	<u>m</u>
	Games-Howell
	Tamhane T2
	Scheffe S
-	Brown-Forsythe /52
	Newman-Keuls

Multiple Comparison Procedures

IBM SPSS Knowledge Center, http://www.ibm.com/support/knowledgecenter/SSLVMB_21.0.0/co m.ibm.spss.statistics.help/idh_onew_post.htm

- Tamhane T2,
- Scheffe S
- Brown-Forsythe
- Newman-Keuls
- Duncan, <u>https://en.wikipedia.org/wiki/Duncan's_new_multiple_range_te</u> <u>st</u>
- Dunnel;s T3

Exhibit 17-14 ANOVA Plots (One-way): Business Class



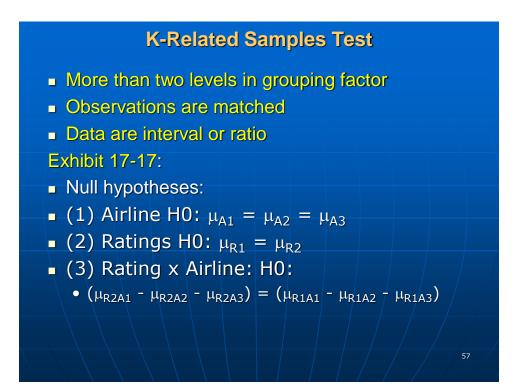
Example 17-15 Two-Way ANOVA Example (Adding Seat Selection: Economy, Business)

- We can now answer three questions:
 - Are differences in flight service ratings attributable to airlines?
 - · Are differences in flight service ratings attributable to seat selection?
 - · Do the airline and seat selections interact with respect to flight service ratings?

 Exhibit 17-15 reports a test of the hypotheses for these 3 questions 								
Model Summary								
Source	d.f.	Sum of Squares	Mean Square	<i>F</i> Value	p Value			
Airline	2	11644.033	5822.017	39.178	0.0001			
Seat selection	1	3182.817	3182.817	21.418	0.0001			
Airline by seat selection	2	517.033	258.517	1.740	0.1853			
Residual	54	8024.700	148.606					

Example 17-15 Two-Way ANOVA Example (Adding Seat Selection: Economy, Business)

	Means Table E					
	Count	Mean	Std. Dev.	Std. Error		
Lufthansa economy	10	35.600	12.140	3.839		
Lufthansa business	10	42.300	15.550	4.917		
Malaysia Airlines economy	10	48.500	12.501	3.953		
Malaysia Airlines business	10	69.300	9.166	2.898		
Cathay Pacific economy	10	64.800	13.037	4.123		
Cathay Pacific business	10	81.000	9.603	3.037		
56						



Example 17-17 Repeated-Measures ANOVA Example

Model Summary							
Source	d.f.	Sum of Squares	Mean Square	<i>F</i> Value	p Value		
Airline	2	3552735.50	17763.775	67.199	0.0001		
Subject (group)	57	15067.650	264.345				
Ratings	1	625.633	625.633	14.318	0.0004		
Ratings by air	2	2061.717	1030.858	23.592	0.0001		
Ratings by subj	57	2490.650	43.696				
					58		

Example 17-17 Repeated-Measures ANOVA Example

Means Table by Airline							
	Count	Mean	Std. Dev.	Std. Error			
Rating 1, Lufthansa	20	38.950	14.006	3.132			
Rating 1, Malaysia Airlines	20	58.900	15.089	3.374			
Rating 1, Cathay Pacific	20	72.900	13.902	3.108			
Rating 2, Lufthansa	20	32.400	8.268	1.849			
Rating 2, Malaysia Airlines	20	72.250	10.572	2.364			
Rating 2, Cathay Pacific	20	79.800	11.265	2.519			

Example 17-17 Repeated-Measures ANOVA Example

Means Table Effect: Ratings							
	Count	Mean	Std. Dev.	Std. Error			
Rating 1	60	56.917	19.902	2.569			
Rating 2	60	61.483	23.208	2.996			
				60			

